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Propositional inquisitive logic: a survey

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Abstract

This paper provides a concise survey of a body of recent work on propositional inquisitive logic. We review the conceptual foundations of inquisitive semantics, introduce the propositional system, discuss its relations with classical, intuitionistic, and dependence logic, and describe an important feature of inquisitive proofs.

Keywords: questions, inquisitive logic, dependency, intermediate logics, proofs-as-programs.

1 Introduction

Inquisitive semantics stems from a line of work which, going back to [12], has aimed at providing a uniform semantic foundation for the interpretation of both statements and questions. The approach was developed in an early version, based on pairs of models, in [13, 16]; it reached the present form, based on information states, in [3, 9], where the associated propositional logic was also investigated. An algebraic underpinning for the inquisitive treatment of logical operators was given in [19]. The foundations of the inquisitive approach have been motivated starting from a language-oriented perspective in [11], and starting from logic-oriented perspective in [7, 8].

The aim of this paper is to provide a short survey of the work done on propositional inquisitive logic, drawing mostly on [3, 9, 6, 8]. More precise pointers to the literature will be provided when discussing specific topics. We will start in Section 2 by showing at a general level how questions can be brought within the scope of logic by means of a simple but fundamental shift in the way semantics is viewed. In Section 3, we instantiate this general approach in the propositional setting, introducing propositional inquisitive logic. In Sections 4, 5, and 6, we examine the connections of this logic to the propositional versions of classical logic, intuitionistic logic, and dependence logic. In Section 7 we discuss inquisitive proofs and their constructive content. In Section 8, we present an extension and a generalization of propositional inquisitive logic. Section 9 wraps up and concludes.

2 Bringing question into the logical landscape

Traditionally, logical entailment captures relations such as the one exemplified by (1): the information that Alice and Bob live in the same city, combined with the information that Alice lives in Amsterdam, yields the information that Bob lives in Amsterdam.

(1) Alice and Bob live in the same city
Alice lives in Amsterdam
———————————————
Bob lives in Amsterdam

Inquisitive logic brings questions into this standard picture, broadening the notion of entailment so as to encompass patterns which we might write as in (2): the information that Alice and Bob live in the same city, combined with the information on where Alice lives, yields the information on where Bob lives.

(2) Alice and Bob live in the same city
Where Alice lives
———————————————
Where Bob lives

Notice the crucial difference between the two examples: in (1) we are concerned with a relation holding between three specific pieces of information. The situation is different in (2): given the information that

Alice and Bob live in the same city, *any* given piece of information on Alice's city of residence yields some corresponding information on Bob's city of residence. We may say that what is at play in (2) are two *types* of information, which we may see as labeled by the questions *where Alice lives* and *where Bob lives*. Entailment captures the fact that, given the assumption that Alice and Bob live in the same city, information of the first type yields information of the second type.

The entrance of questions into the logical arena is made possible by a fundamental shift in the way the semantics of a sentence is construed. In classical logic, the meaning of a sentence is given by laying out in what states of affairs the sentence is *true*; however, this truth-conditional view does not seem suitable in the case of questions. In inquisitive logic, by contrast, the meaning of a sentence is given by laying out what information is needed in order for a sentence to be *supported*. Accordingly, sentences are evaluated relative to objects called *information states*, which formally encode bodies of information.

Unlike truth-conditional approach, the support approach is applicable to both statements and questions. To give concrete examples, a statement like (3-a) is supported by an information state s if the information available in s implies that Alice lives in Amsterdam; on the other hand, a question like (3-b) is supported by an information state s if the information available in s determines where Alice lives.

(3) a. Alice lives in Amsterdam.
b. Where does Alice live?

This more general semantic approach comes with a corresponding notion of entailment, understood as preservation of support: an entailment holds if the conclusion is supported whenever all the premises are. Assuming a natural connection between the truth-conditions of a statement and its support conditions—namely, that a state supports a statement iff it implies that the statement is true—this notion of entailment coincides with the truth-conditional one as far as statements are concerned. The novelty, however, lies in the fact that now, questions can also participate in entailment relations. Thus, for example, we can indeed capture the pattern in (2) as a case of logical entailment. To see this, suppose an information state s supports the premises of (2): this means that the information available in s implies that Alice and Bob live in the same city, and also determines in which city Alice lives; clearly, then, the information available in the state determines in which city Bob lives, which means that the conclusion of (2) is supported.

The one discussed in this section is a very general approach to logic, which can be instantiated by a range of concrete systems, differing with respect to their logical language and to the relevant notion of information states. Just as for classical logic, we have inquisitive logics of different sorts: propositional, modal, first-order, etc. The remaining sections of the paper provide an overview of the results obtained in the most basic and best understood setting—the propositional one.[1,2]

3 Propositional inquisitive logic

The language of propositional inquisitive logic, InqB, is the propositional language built up from a set of atomic sentences and $\bot$ by means of conjunction, $\land$, implication, $\to$, and inquisitive disjunction, $\mathbin{\vee\!\!\!\vee}$.

$$\phi \ ::= \ p \mid \bot \mid \phi \land \phi \mid \phi \to \phi \mid \phi \mathbin{\vee\!\!\!\vee} \phi$$

Negation and classical disjunction are defined by setting $\neg\phi := \phi \to \bot$, and $\phi \lor \psi := \neg(\neg\phi \land \neg\psi)$. Formulas that contain no occurrence of $\mathbin{\vee\!\!\!\vee}$ are called *classical* formulas.

In the propositional setting, an information state is construed as a set of propositional valuations. The idea here is that a set s encodes the information that the actual state of affairs corresponds to one of the valuations in s. This means that if $t \subseteq s$, then t contains at least as much information as s, and possibly more.

[1] For discussion on the semantic foundations of the inquisitive approach, on the role of questions in logic, and on the relation between truth and support, see [6, 8].

[2] The research in inquisitive modal logic and inquisitive first-order logic has also been growing rapidly in these last few years. Recent work includes [10, 4, 6, 22].

The clauses defining the relation of *support* relative to an information state are the following ones:

- $s \models p \iff w(p) = 1$ for all $w \in s$
- $s \models \bot \iff s = \emptyset$
- $s \models \phi \land \psi \iff s \models \phi$ and $s \models \psi$
- $s \models \phi \mathbin{\vee\!\!\!\vee} \psi \iff s \models \phi$ or $s \models \psi$
- $s \models \phi \to \psi \iff \forall t \subseteq s : t \models \phi$ implies $t \models \psi$.

A key feature of the semantics is *persistency*: if ϕ is supported by an information state s, then it is also supported by any state $t \subseteq s$ which contains at least as much information. This means that as information grows, more and more formulas become supported. In the information state $\emptyset$, which represents the state of *inconsistent* information, every formula is supported. This may be regarded as a semantic analogue of the *ex falso quodlibet* principle.

4 Relations with classical logic

In inquisitive logic, the fundamental semantic notion is that of support relative to an information state. However, the notion of *truth* relative to a particular valuation w can be recovered by setting: $w \models \phi \iff \{w\} \models \phi$. It is then easy to check that all classical formulas receive the standard truth-conditions.

For some formulas, support at a state simply amounts to truth at each world in the state. If this is the case, we say that the formula is *truth-conditional*. More formally, ϕ is truth-conditional in case for all states s: $s \models \phi \iff \forall w \in s, w \models \phi$. We regard truth-conditional formulas as corresponding to *statements*. The intuition is that there is only one way for an information state s to support a statement: the information available in s must imply that the statement is true.

As a matter of fact, large classes of formulas in InqB are truth-conditional. In particular, all classical formulas are.

Proposition 1. All classical formulas are truth-conditional.

This means that all classical formulas receive essentially the same treatment as in classical propositional logic: their semantics is fully determined by their truth-conditions, which in turn are the standard ones. This is reflected by the relation of entailment among these formulas.

Proposition 2 (Conservativity over classical logic)**.**
Entailment restricted to classical formulas coincides with entailment in classical propositional logic.

This means that the classical fragment of InqB can be identified for all intents and purposes with classical propositional logic, and our logic may be regarded as a conservative extension of classical propositional logic with an inquisitive disjunction operator.

Formulas formed by means of inquisitive disjunction are typically not truth-conditional. We take such formulas to correspond to *questions.* For instance, the formula $p \vee\!\!\!\vee \neg p$, abbreviated as $?p$, corresponds to the question *whether p or not p.* An information state can support this formula in two different ways: either by implying that p is true, or by implying that p is false. Similarly, the formula $p \vee\!\!\!\vee q$ can be regarded as encoding the question *whether p or q*, which can be supported either by establishing that p is true, or by establishing that q is true.[3]

Like in classical logic, formulas in inquisitive logic can be written in a very constrained normal form: namely, any formula of InqB can be written as an inquisitive disjunction of classical formulas.

Theorem 1 (Inquisitive normal form)**.**
Recursively on ϕ, we can define a set $\mathcal{R}(\phi) = \{\alpha_1, \ldots, \alpha_n\}$ of classical formulas, called the *resolutions* of ϕ, such that $\phi \equiv \alpha_1 \vee\!\!\!\vee \ldots \vee\!\!\!\vee \alpha_n$.

Intuitively, we can regard the resolutions of a formula as capturing the different ways in which the formula may be supported. If ϕ is a

[3] An exclusive reading of the question *whether p or q* can be formalized as well, by translating the question as $(p \wedge \neg q) \vee\!\!\!\vee (q \wedge \neg p)$.

classical formula, then it can be supported in only one way, by establishing that it is true; accordingly, we have $\mathcal{R}(\phi) = \{\phi\}$. On the other hand, if ϕ stands for a question, there will be multiple ways of supporting the formula, and thus multiple resolutions; for instance, we have $\mathcal{R}(?p) = \{p, \neg p\}$, and $\mathcal{R}(p \mathbin{\vee\!\!\!\vee} q) = \{p, q\}$. Any formula in InqB can thus be construed as offering a (possibly trivial) choice among classical formulas.

We saw that entailments among classical formulas amount to entailments in classical logic. On the other hand, we saw that our language also includes formulas which can be regarded as questions. Instances of entailment which involve such formulas capture interesting logical relations that lack a counterpart in classical logic; notably, entailments involving both question assumptions and question conclusions capture relations of *logical dependency* among these questions, possibly within the context of certain statements. For instance, the following entailment captures the fact that, given the information that $r \leftrightarrow p \wedge q$, the question $?r$ is completely determined by the questions $?p$ and $?q$.

$$r \leftrightarrow p \wedge q,\ ?p,\ ?q \models ?r$$

Summing up, then, propositional inquisitive logic can be regarded as a conservative extension of classical propositional logic with an inquisitive disjunction: while the classical fragment of the language coincides with classical logic, by means of the operator $\mathbin{\vee\!\!\!\vee}$ we can build formulas which express propositional questions, and capture dependencies among such questions as special cases of the relation of entailment.[4]

5 Relations with intuitionistic logic

In the previous section, we saw that InqB can be viewed as a conservative extension of classical logic if $\mathbin{\vee\!\!\!\vee}$ is regarded as an additional, non-standard connective. In this section we will show that if, on the other hand, we regard $\mathbin{\vee\!\!\!\vee}$ as the standard disjunction of the system,

[4] For more on the relations between inquisitive logic and classical logic, see [6, 8].

then InqB turns out to be a special kind of *intermediate* logic, i.e., a logic sitting in between intuitionistic and classical logic.

The first step in this direction is to notice that our semantics can be regarded as a case of intuitionistic Kripke semantics on a particular Kripke model, having consistent information states as its elements, the relation $\supseteq$ as accessibility relation, and the valuation function $V(p) = \{s \mid w(p) = 1 \text{ for all } w \in s\}$. Since Kripke semantics is sound for intuitionistic logic, this implies that anything that can be falsified in inquisitive logic can be falsified in intuitionistic propositional logic, IPL. On the other hand, it is easy to see that singleton information states $\{w\}$ behave just like the corresponding propositional valuation w: this ensures that anything that can be falsified in classical logic, CPL, can also be falsified in inquisitive logic. If we identify a logic with the corresponding set of validities, we can sum up our findings as follows.

Proposition 3. $\mathsf{IPL} \subseteq \mathsf{InqB} \subseteq \mathsf{CPL}$

Thus, from this perspective InqB is a logic stronger than intuitionistic logic, but weaker than classical logic. It is not, however, an *intermediate logic* in the usual sense of the term. This is because InqB is not closed under the rule of *uniform substitution*: in particular, the double negation law is valid for propositional atoms, but invalid when atoms are replaced by questions: $\neg\neg p \to p \in \mathsf{InqB}$, but $\neg\neg ?p \to ?p \notin \mathsf{InqB}$. The conceptual point here is that atoms in InqB are not intended as placeholders for arbitrary sentences, but only placeholders for arbitrary *statements*. As we saw, statements are truth-conditional, and as such they validate the double negation law, which is not generally valid. It is worth emphasizing that this is not an accident, but a deliberate architectural choice (see pp. 66-67 of [6]). This choice (i) enables InqB to retain a classical fragment, which encodes the underlying logic of statements; (ii) allows for a recursive decomposition of questions into resolutions; and (iii) makes a neat proof system possible.

Besides this classical feature of atoms, inquisitive logic differs from intuitionistic logic in that the space of information states has a special structure, which renders valid some non-intuitionistic principles. The best known of these is the Kreisel-Putnam scheme, first studied in [14]:

$$(\mathsf{KP}) \qquad (\neg\phi \to \psi \mathbin{\vee\!\!\!\vee} \chi) \to (\neg\phi \to \psi) \mathbin{\vee\!\!\!\vee} (\neg\phi \to \chi)$$

While this principle may look mysterious at first, it can be shown (see p. 80 of [6]) to encode a fundamental relation between statements and questions: a statement only counts as resolving a question if it entails a specific resolution to the question.

As shown in [9], the classicality of atoms and the validity of the KP scheme, together with the underlying intuitionistic base, suffice to characterize inquisitive propositional logic completely. More formally, InqB can be characterized as the set of formulas obtained by extending IPL with all instances of KP and with $\neg\neg p \to p$ for all atoms p, and closing the resulting set under *modus ponens*.

Theorem 2. $\mathsf{InqB} = \mathsf{IPL} + \mathsf{KP} + \neg\neg p \to p$

In fact, besides the Kreisel-Putnam logic axiomatized by the scheme KP, there is a whole range of intermediate logics which, when extended with classical atoms, yield inquisitive logic: as shown in [9], this range consists exactly of those intermediate logics which include Maksimova's logic [15] and are included in Medvedev's logic of finite problems [17, 18]. In particular, Medvedev's logic is the largest standard intermediate logic included in InqB.

An important aspect of the relation between inquisitive logic and intuitionistic logic can be observed based on the normal form result given by Theorem 1. This result guarantees that any formula can be written as an inquisitive disjunction of classical formulas. Since classical formulas behave as in classical logic, they are logically equivalent to their own double negation. Thus, it follows that in InqB, any formula ϕ is equivalent to an inquisitive disjunction of negations $\phi^{\mathsf{DNT}} = \neg\psi_1 \mathbin{\vee\!\!\!\vee} \ldots \mathbin{\vee\!\!\!\vee} \neg\psi_n$. Now, the following theorem shows that the map $(\cdot)^{\mathsf{DNT}}$ is a translation of inquisitive logic into intuitionistic logic.

Theorem 3. $\Phi \models \psi \iff \Phi^{\mathsf{DNT}} \models_{\mathsf{IPL}} \psi^{\mathsf{DNT}}$

This result can be extended to show that the Lindenbaum-Tarski algebra for InqB is isomorphic to the sub-algebra of the Lindenbaum-Tarski algebra for IPL consisting of equivalence classes of disjunctions of negations. Thus, while classical propositional logic can be regarded as the negative fragment of intuitionistic logic, propositional inquisitive logic can be regarded as the *disjunctive-negative fragment* of intuitionistic logic—the fragment consisting of disjunctions of negations.[5]

6 Relations with dependence logic

We mentioned above that in inquisitive logic, entailments involving questions capture logical dependencies. The relation of dependency is also the focus of recent work in the framework of *dependence logic* [23]. Dependence logic and inquisitive logic are tightly connected frameworks, as discussed in detail in [7]. In the propositional setting, full translations are possible between the two [25]. In both propositional systems, formulas are interpreted relative to sets of assignments; while propositional inquisitive logic enriches classical propositional logic with questions, propositional dependence logic enriches it with formulas called *dependence atoms*, written $=(p_1, \ldots, p_n, q)$, which capture the fact that the truth-value of an atomic proposition q is determined by the truth-values of other atomic propositions $p_1, \ldots, p_n$. The semantics of these atoms is given by the following clause:

$$s \models =(p_1, \ldots, p_n, q) \iff \forall w, w' \in s: \text{ if } w(p_i) = w'(p_i) \text{ for all } i, \text{ then } w(q) = w'(q)$$

It is easy to check that such a dependence atom can be expressed in InqB by means of the formula $?p_1 \wedge \cdots \wedge ?p_n \to ?q$. This is not an accident: as shown in [7], in inquisitive logic, the fact that a question ν is fully determined by questions $\mu_1, \ldots, \mu_n$ is generally captured by the implication $\mu_1 \wedge \cdots \wedge \mu_n \to \nu$. More precisely, the formula $\mu_1 \wedge \cdots \wedge \mu_n \to$

[5] For more on the relations between propositional inquisitive logic, intuitionistic logic, and intermediate logics, see [3] and [9].

ν is supported at a state s in case relative to s, any way of resolving the questions $\mu_1, \ldots, \mu_n$ determines a corresponding way to resolve the question ν. What a dependence atom expresses is that the question $?q$ is determined by the questions $?p_1, \ldots, ?p_n$, hence the representation $?p_1 \wedge \cdots \wedge ?p_n \to ?q$.

Realizing that dependencies can be captured generally as implications between questions is interesting for various reasons. The first kind of reason is proof-theoretic: in inquisitive logic, all the connectives, including those involved in a dependence formula, can be handled by essentially standard inference rules. Thus, for instance, a dependency $?p \to ?q$ may be formally proved to hold by assuming the question $?p$ and trying to conclude the question $?q$. In fact, this perspective brings out the fact that the *Armstrong axioms* for functional dependency [2] used in database theory are essentially nothing but the axioms of implication in disguise—a fact that was first noted in [1].

Moreover, realizing that dependencies can be generally captured as implications between questions allows us to see that dependence atoms are a particular case of a more general pattern. Not just for atomic polar questions of the form $?p$, but for all sorts of questions $\mu_1, \ldots, \mu_n, \nu$ expressible in the system—in fact, in *any* inquisitive system—the fact that ν is determined by $\mu_1, \ldots, \mu_n$ is expressed by $\mu_1, \ldots, \mu_n \to \nu$.

Finally, realizing that dependencies can be expressed as implications among questions allows us to use inquisitive logics to investigate the logical properties of the notion of dependency. For example, consider the valid entailment $?p,\ ?p \wedge ?q \to ?r \models ?q \to ?r$. This captures the fact that given the information whether p, from a dependency of $?r$ on both $?p$ and $?q$ we can always compute a dependency of $?r$ on $?q$. If we think of a dependency as encoded by a function (cf. the notion of *dependence function* in §2 of [6]), this amounts to the fact that we can saturate one of the arguments of this function.[6]

[6] For more on the relations between inquisitive and dependence logic, see [7, 6, 26].

7 Questions in proofs

An important feature of inquisitive logic is that it shows that questions can meaningfully be manipulated in logical inferences, and that their logical behavior is in fact rather familiar. In the propositional setting, a natural deduction system for inquisitive logic is obtained by extending a system for intuitionistic logic with the following two inference rules, where α ranges over classical formulas, and ϕ, ψ over arbitrary formulas.

$$\frac{\alpha \to (\phi \mathbin{\vee\!\!\!\vee} \psi)}{(\alpha \to \phi) \mathbin{\vee\!\!\!\vee} (\alpha \to \psi)}\ (\mathsf{split}) \qquad\qquad \frac{\neg\neg\alpha}{\alpha}\ (\mathsf{dne})$$

The second of these rules captures the fact that classical formulas are truth-conditional, and thus behave exactly as in classical logic. The first—related to the Kreisel-Putnam scheme discussed above—captures the interaction among statements and questions, stipulating that if a statement resolves a question, it must do so by yielding a particular resolution to it. The completeness of this system for InqB, proved in [6], implies in particular that any valid propositional dependency can be formally proved by making inferences with propositional questions in this system. Thus, questions are interesting proof-theoretic tools: by making inferences with them, we can establish the existence of certain logical dependencies. Moreover, the following theorem, proved in [6], shows that a proof of a dependency does not just *witness* that the dependency holds, but actually encodes a method for computing it.

Theorem 4 (Constructive content of inquisitive proofs).
Suppose P is a natural deduction proof having assumptions $\phi_1, \dots, \phi_n$ and conclusion ψ. Recursively on P, we can define a procedure f_P which, when given as input resolutions $\alpha_1, \dots, \alpha_n$ of the assumptions, outputs a resolution $f_P(\alpha_1, \dots, \alpha_n)$ of the conclusion with the property that $\alpha_1, \dots, \alpha_n \models f_P(\alpha_1, \dots, \alpha_n)$.

What this theorem shows is that proofs in inquisitive logic have a specific kind of constructive content: they encode methods for turning any given resolutions of the question assumptions into a resolution of the conclusion which is determined by them. This is reminiscent of the proofs-as-programs interpretation of intuitionstic logic, and it shows once more that, while our logic coincides with classical logic on statements, encoded by classical formulas, the logic of questions has a constructive flavor to it.[7]

8 Extensions and generalization

In the last couple of years, the work on propositional inquisitive logic presented in the previous sections has been extended in several directions. First of all, it has been taken as the basis for logics that go beyond the propositional realm, such as the modal logics given in [4, 10, 6], and the first-order logics given in [6, 7]. Presenting these richer logics goes beyond the scope of the present survey. However, in this section I want to briefly discuss an extension and a generalization of InqB, both due to Vít Punčochář, that remain within the domain of propositional logic.

First, the system InqB is extended in [20] with a *weak negation* connective, denoted $\sim$, which allows us to express the fact that a certain formula fails to be supported at the evaluation state.

$$s \models \sim\phi \iff s \not\models \phi$$

Evidently, the addition of this connective results in a system in which support is no longer persistent: a formula $\sim\phi$ may be supported by a state s, yet it may fail to be supported by a stronger state $t \subseteq s$. One reason why such a system is interesting is that—while remaining within the propositional inquisitive setting—it allows for the definition of formulas $\Diamond\phi$ which express the fact that the state of evaluation can be extended consistently to support ϕ. Interestingly, this logic is axiomatized by means of a proof system which allows for two different modes of hypothetical proofs. In one mode, making the assumption ϕ corresponds to supposing that the current information state supports ϕ. In

[7] For more on the role of questions in inference and on the constructive content of inquisitive proofs, see [8, 6, 5].

the other mode, it corresponds to supposing that the current information state is extended so as to support ϕ. In this second mode, only some formulas from outside the hypothetical context can be appealed to when reasoning within the hypothetical context.

A generalization of propositional inquisitive logic is explored in [21]. This paper defines an operation G which, given a logic Λ with $\mathsf{IPL} \subseteq \Lambda \subseteq \mathsf{CPL}$, returns a corresponding logic $\mathrm{G}(\Lambda)$, called the *global variant* of Λ. Logics of the form $\mathrm{G}(\Lambda)$ are called G-logics.[8] Intuitively, $\mathrm{G}(\Lambda)$ is a logic obtained by extending the $\mathbin{\rotatebox[origin=c]{-90}{$\geqslant$}}$-free fragment of Λ with an inquisitive disjunction connective. In Section 4, we saw that InqB can be seen as arising from extending classical logic with inquisitive disjunction. And indeed, we have $\mathsf{InqB} = \mathrm{G}(\mathsf{CPL})$, which means that inquisitive logic is the greatest of all G-logics. The smallest G-logic, $\mathrm{G}(\mathsf{IPL})$, is the logic $\mathsf{IPL}{+}\mathsf{H}$ axiomatized by extending intuitionistic logic with the following scheme, where ϕ, ψ range over arbitrary formulas, and α ranges over Harrop formulas, and closing under *modus ponens*.[9]

$$(\mathsf{H}) \qquad (\alpha \to \phi \mathbin{\rotatebox[origin=c]{-90}{$\geqslant$}} \psi) \to (\alpha \to \phi) \mathbin{\rotatebox[origin=c]{-90}{$\geqslant$}} (\alpha \to \psi)$$

All other G-logics fall in between $\mathsf{IPL}+\mathsf{H}$ and InqB, and share many of the core features of inquisitive logic. All of them have the disjunction property, meaning that a disjunction $\phi \mathbin{\rotatebox[origin=c]{-90}{$\geqslant$}} \psi$ can only be valid if either ϕ or ψ is valid. None of them is closed under uniform substitution. All of them coincide with the base logic Λ in their $\mathbin{\rotatebox[origin=c]{-90}{$\geqslant$}}$-free fragment, and allow for an analogue of Theorem 1, stating that any formula is equivalent to a disjunction $\alpha_1 \mathbin{\rotatebox[origin=c]{-90}{$\geqslant$}} \ldots \mathbin{\rotatebox[origin=c]{-90}{$\geqslant$}} \alpha_n$ of classical formulas. Finally, all G-logics can be characterized axiomatically in a uniform way: $\mathrm{G}(\Lambda)$ amounts to the logic obtained by extending intuitionistic logic with the scheme H and all $\mathbin{\rotatebox[origin=c]{-90}{$\geqslant$}}$-free formulas which are valid in Λ, and closing this set under *modus ponens*.

[8] Here, the logic Λ is assumed to be closed under *modus ponens*, but not necessarily under uniform substitution.

[9] A Harrop formula is defined as a formula in which disjunction is only allowed to occur within the antecedent of an implication.

9 Conclusion

In this paper I have tried to give a bird's eye view of propositional inquisitive logic, including its conceptual underpinnings, its main mathematical features, and its relations to other logics. My hope is that this survey, together with the pointers scattered through the paper, will provide a valuable guide to the growing literature on the subject.

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