Inquisitive semantics: a new notion of meaning

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March 26, 2013

Abstract
This paper presents a notion of meaning that captures both informative and inquisitive content, which forms the cornerstone of inquisitive semantics. The new notion of meaning is explained and motivated in detail, and compared to previous inquisitive notions of meaning.

1 Introduction

Recent work on inquisitive semantics has given rise to a new notion of meaning, which captures both informative and inquisitive content in an integrated way. This enriched notion of meaning generalizes the classical, truth-conditional notion of meaning, and provides new foundations for the analysis of linguistic discourse that is aimed at exchanging information.

The way in which inquisitive semantics enriches the notion of meaning changes our perspective on logic as well. Besides the classical notion of entailment, the semantics also gives rise to a new notion of inquisitive entailment, and to new logical notions of relatedness, which determine, for instance, whether a sentence compliably addresses or resolves a given issue.

The enriched notion of semantic meaning also changes our perspective on pragmatics. The general objective of pragmatics is to explain aspects of interpretation that are not directly dictated by semantic content, in terms of general features of rational human behavior. Gricean pragmatics has fruitfully pursued this general objective, but is limited in scope. Namely, it is only concerned with what it means for speakers to behave rationally in providing information. Inquisitive pragmatics is broader in scope: it is both speaker- and hearer-oriented, and is concerned more generally with what it means to behave rationally in cooperatively exchanging information rather than just in providing information. This makes it possible to derive a wider range of implicatures, in particular ones that arise from inquisitiveness.

This paper focuses on the notion of meaning that underlies the most basic implementation of inquisitive semantics. Our aim here is to explain and motivate this new notion of meaning in detail, and to place it in a wider context of previous theories about inquisitive aspects of meaning. Other work (e.g. Ciardelli, 2009; Ciardelli and Roelofsen, 2011; Groenendijk and Roelofsen, 2009; Roelof- sen, 2011; AnderBois, 2011; Pruitt and Roelofsen, 2011; Farkas and Roelofsen,
2012) discusses how this new type of meanings could be assigned to expressions in formal and simple fragments of natural languages, and explores some of the logical and pragmatic repercussions of this shift in perspective. The current paper can be seen as a retrospective preface to this body of work, aiming to provide a detailed specification and principled motivation for its basic notions.

The paper is organized as follows. The new inquisitive notion of meaning is first characterized in general terms in section 2, and then fleshed out in more detail in sections 3 and 4. Section 5 illustrates how the resulting framework can be used to capture the meaning of various types of sentences in natural language, and finally, section 6 situates our proposal in the context of previous work on inquisitive aspects of meaning.

2 Meaning as information exchange potential

The meaning of a sentence can be thought of as something that determines the intended effect of an utterance of that sentence on the discourse context in which the sentence is uttered. That is, when a speaker utters a sentence in a certain discourse context, he intends his utterance to change the discourse context in a particular way, and the meaning of the sentence determines this intended effect. Thus, the meaning of a sentence can be conceived of as its context change potential, which can be modeled formally as a function that maps every discourse context to a new discourse context.

This general conception of meaning can be made more precise in several ways, depending on what exactly we take a discourse context to be. The simplest and most common option is to think of a discourse context as the body of information that has been established in the discourse so far. This body of information is usually referred to as the common ground of the conversation, and it is formally modeled as a set of possible worlds—those worlds that are compatible with the established information.

If the discourse context is identified with the information established so far, then the context change potential of a sentence boils down to its information change potential. Formally, the meaning of a sentence can then be modeled as a function that maps information states—sets of possible worlds—to other information states.

Classically, the meaning of a sentence is identified with its truth-conditions, rather than a function over information states. However, a specific connection may be assumed between the truth-conditions of a sentence and the intended effect of uttering that sentence in a certain discourse context. Namely, it may be assumed that the intended effect of uttering a sentence in a certain discourse context is to restrict that discourse context to precisely those worlds that satisfy the truth-conditions of the uttered sentence, i.e., to those worlds in which the sentence is true. Under this assumption, the truth-conditions of a sentence completely determine the sentence’s information change potential. In light of this connection, the classical truth-conditional framework can also be seen as one in which the meaning of a sentence is something that determines the sentence’s
context change potential.

Now, as noted above, identifying a discourse context with the body of information that has been established in the discourse so far, is only one particular way of spelling out the notion of a discourse context. Of course, the general conception of meaning as context change potential is in principle compatible with richer notions of discourse contexts. And to analyze many types of discourse, such richer notions are indeed required. We will focus here on a very basic type of discourse, namely one in which a number of participants exchange information by raising and resolving issues. In order to analyze this type of discourse, discourse contexts should not only embody the information that has been established so far, but also the issues that have been raised so far. And similarly, the meaning of a sentence should not only embody its informative content, i.e., its potential to provide information, but also its inquisitive content, i.e., its potential to raise issues. In short, the meaning of a sentence should embody its information exchange potential. Below we will spell out in detail how to model such richer types of discourse contexts and meanings.

3 Discourse contexts: information and issues

The first step is to formulate a notion of discourse contexts that embodies both the information that has been established so far and the issues that have been raised so far. We already know how we can model the information established so far, namely as a set $s$ of possible worlds. Throughout the paper we will assume a set of possible worlds $\omega$ as our logical space.

**Definition 1 (Information states).**

An information state $s$ is a set of possible worlds, i.e., $s \subseteq \omega$.

We will often refer to information states simply as *states*, and for any discourse context $c$ we will use $\text{info}_c$ to denote the information state that represents the information available in $c$. The crucial question that remains to be addressed is how to model *issues*.

**Issues.** Suppose we are in a discourse context $c$. Then, according to the information established so far, the actual world is located somewhere in $\text{info}_c$. We can think of every subset $s \subseteq \text{info}_c$ as an information state that is at least as informed as $\text{info}_c$, i.e., one that locates the actual world at least as precisely as $\text{info}_c$ itself. We thus refer to every state $s \subseteq \text{info}_c$ as a possible *enhancement* of $\text{info}_c$.\(^1\)

\(^1\)This deliberately includes $\text{info}_c$, which is thought of as a trivial enhancement of itself.

Now, an issue in $c$ should represent a certain request for information, a request to locate the actual world more precisely inside $\text{info}_c$. Thus, an issue in $c$ can be modeled as a non-empty set $I$ of enhancements of $\text{info}_c$, namely those enhancements that would satisfy the given request, i.e., that would locate the actual world with sufficient precision.
Importantly, if we take this perspective, not just any non-empty set $I$ of enhancements of $\text{info}_c$ can properly be thought of as an issue in $c$. First, if $I$ contains a certain enhancement $s$ of $\text{info}_c$, and $t \subset s$ is a further enhancement of $s$, then $t$ must also be in $I$. After all, if $s$ locates the actual world with sufficient precision, then $t$ cannot fail to do so as well. So $I$ must be downward closed.

Second, the elements of $I$ must together form a cover of $\text{info}_c$. That is, every world in $\text{info}_c$ must be included in at least one element of $I$. After all, any world in $\text{info}_c$ may be the actual world according to the information available in $c$. Now suppose that $w$ is a world in $\text{info}_c$ that is not included in any element of $I$. Then, according to the information available in $c$, $w$ may very well be the actual world. But if it is indeed the actual world, then it would be impossible to satisfy the request represented by $I$ without discarding the actual world. Thus, in order to ensure that it is possible to satisfy the request represented by $I$ without discarding the actual world, $I$ should form a cover of $\text{info}_c$. This leads us to the following notion of an issue.

**Definition 2 (Issues).**
Let $s$ be an information state, and $I$ a non-empty set of enhancements of $s$. Then we say that $I$ is an issue over $s$ if and only if:

1. $I$ is downward closed: if $t \in I$ and $t' \subset t$ then also $t' \in I$
2. $I$ forms a cover of $s$: $\bigcup I = s$

**Definition 3 (Settling an issue).**
Let $s$ be an information state, $t$ an enhancement of $s$, and $I$ an issue over $s$. Then we say that $t$ settles $I$ if and only if $t \in I$.

Notice that our definition of an issue over $s$ includes as a limit case the issue $\wp(\text{info}_c)$ consisting of all enhancements of $s$. We refer to this issue, which is already settled by the information available in $s$, as the trivial issue over $s$, and we regard it as encoding the absence of a request for information.

Equipped with a precise definition of issues, we are now ready to return to the notion of a discourse context, our main concern in this section.

**Discourse contexts.** Given what we have said so far, the most straightforward way to proceed would be to define a discourse context $c$ as a pair $\langle \text{info}_c, \text{issues}_c \rangle$, where $\text{info}_c$ is an information state, and $\text{issues}_c$ a set of issues over $\text{info}_c$. The initial discourse context would then be $\langle \omega, \emptyset \rangle$, consisting of the trivial information state, which does not rule out any world, and the empty set of issues.

This would indeed be a suitable notion of discourse contexts. However, for our current purposes, it will be convenient to simplify this notion somewhat. We will do this in two steps. First, rather than thinking of a discourse context $c$ as a pair $\langle \text{info}_c, \text{issues}_c \rangle$ where $\text{issues}_c$ is a set of issues over $\text{info}_c$, we could think of a discourse context as a pair $\langle \text{info}_c, \text{issue}_c \rangle$ where $\text{issue}_c$ is a single issue over $\text{info}_c$. This simplification is justified by the observation that any set of issues
I over a state $s$ can be merged into a single issue $I_I = \{t \subseteq s \mid t \in J \text{ for each } J \in I\}$, which is settled precisely by those enhancements $t \subseteq s$ that settle all issues in $I$. Notice that for $I \neq \emptyset$ the issue $I_I$ amounts to the intersection $\bigcap I$ of all issues in $I$, whereas for $I = \emptyset$ the issue $I_I$ coincides with the trivial issue $\emptyset(s)$ over $s$, which requests no information.$^{2,3}$

So we can think of a discourse context $c$ as a pair $\langle \text{info}_c, \text{issue}_c \rangle$, where $\text{info}_c$ is an information state, and $\text{issue}_c$ a single issue over $\text{info}_c$, and we can take the initial context to be the pair $\langle \omega, \emptyset(\omega) \rangle$, consisting of the trivial information state, which does not rule out any world, and the trivial issue over this state, which is settled by any state (and thus requests no information). But this representation can be simplified even further. After all, since $\text{issue}_c$ is an issue over $\text{info}_c$, it must form a cover of $\text{info}_c$. So we always have that $\text{info}_c = \bigcup \text{issue}_c$. That is, $\text{info}_c$ can always be retrieved from $\text{issue}_c$. But then $\text{info}_c$ can just as well be left out of the representation of $c$. Thus, a discourse context $c$ can simply be represented as an issue over some information state $s$, i.e., a non-empty, downward closed set of enhancements of $s$ that together form a cover of $s$. This information state $s$ is then understood to embody the information available in $c$.

**Definition 4 (Discourse contexts).**

- A discourse context $c$ is a non-empty, downward closed set of states.
- The set of all discourse contexts will be denoted by $\mathcal{C}$.

**Definition 5 (The information available in a discourse context).**

- For any discourse context $c$: $\text{info}_c := \bigcup c$

We have moved from the classical notion of a discourse context as a set of possible worlds—representing the information established so far—to a richer notion of discourse contexts as non-empty, downward closed sets of states—representing both the information established so far and the issues raised so far. With this enriched notion of discourse contexts, we are in principle ready to return to our main concern, which is to specify a notion of *meaning* that embodies both informative and inquisitive content. However, before turning to meanings, it will be useful to briefly identify some special properties that discourse contexts may have.

First of all, we can make a distinction between *informed* and *ignorant* discourse contexts, ones in which some information has been established and ones in which no information has been established yet, respectively.$^{2}$

$^2$For the informative component of a discourse context we have implicitly assumed a similar reduction: we do not keep track of all the separate pieces of information that have been established in the discourse so far, but rather of the set of worlds that are compatible with all these pieces of information (formally, this is again the intersection of all the separate established pieces of information). For certain purposes it may be convenient, or even necessary, to keep track of all the separate pieces of information and/or issues that have been established/raised in a discourse, cf., Stalnaker’s (1978) distinction between the common ground and the context set. However, for our current purposes, this would only add unnecessary complexity.

$^3$Notice that $I_I$ is guaranteed to be an issue in the sense of definition 2. In particular, it is guaranteed to be non-empty, since it will contain the empty state, which settles every issue.
Definition 6 (Informed and ignorant discourse contexts).

- A discourse context \( c \) is informed iff \( \text{info}_c \neq \omega \).
- A discourse context \( c \) is ignorant iff \( \text{info}_c = \omega \).

Similarly, we can make a distinction between inquisitive and indifferent discourse contexts. A discourse context \( c \) is indifferent iff the information that has been established so far settles all the issues that have been raised, i.e., \( \text{info}_c \in c \). Otherwise, i.e., if there are unresolved issues, then \( c \) is called inquisitive.

Definition 7 (Inquisitive and indifferent discourse contexts).

- A discourse context \( c \) is indifferent iff \( \text{info}_c \in c \).
- A discourse context \( c \) is inquisitive iff \( \text{info}_c \notin c \).

There are two special discourse contexts: the initial and the absurd discourse context. The initial context, \( c_{\top} := \wp(\omega) \), is the only context that is both ignorant and indifferent. The absurd context, \( c_{\bot} := \{\emptyset\} \), is one in which the established information is inconsistent and therefore rules out all possible worlds.

Definition 8 (The initial and the absurd discourse context).

- \( c_{\top} := \wp(\omega) \)
- \( c_{\bot} := \{\emptyset\} \)

Two discourse contexts can be compared in terms of the information that has been established or in terms of the issues that have been raised. One context \( c' \) is at least as informed as another context \( c \) if and only if \( \text{info}_{c'} \subseteq \text{info}_c \).

Definition 9 (Informative order on discourse contexts). Let \( c, c' \in \mathcal{C} \). Then:

- \( c' \geq_{\text{info}} c \) iff \( \text{info}_{c'} \subseteq \text{info}_c \)

Similarly, for any two discourse contexts \( c \) and \( c' \) that are equally informed, i.e., \( \text{info}_c = \text{info}_{c'} \), we can say that \( c' \) is at least as inquisitive as \( c \) if and only if every state that settles all the issues that have been raised in \( c' \) also settles all the issues that have been raised in \( c \), i.e., if and only if \( c' \subseteq c \).

Definition 10 (Inquisitive order on discourse contexts). Let \( c, c' \in \mathcal{C} \) and \( \text{info}_c = \text{info}_{c'} \). Then:

- \( c' \geq_{\text{inq}} c \) iff \( c' \subseteq c \)

We will say that one context \( c' \) is an extension of another context \( c \) just in case (i) \( c' \) is at least as informed as \( c \), and (ii) \( c' \) is at least as inquisitive as the context that is obtained by restricting \( c \) to \( \text{info}_{c'} \), i.e., \( c' \geq_{\text{inq}} c \mid \text{info}_{c'} \), where \( c \mid \text{info}_{c'} := \{ s \cap \text{info}_{c'} \mid s \in c \} \). It can be shown that these two conditions are fulfilled just in case \( c' \subseteq c \). So the extension relation between discourse contexts can simply be defined in terms of inclusion.
Definition 11 (Extending discourse contexts). Let \( c, c' \in \mathcal{C} \). Then:

- \( c' \) is an extension of \( c \), \( c' \geq c \), iff \( c' \subseteq c \)

The extension relation forms a partial order on \( \mathcal{C} \), and \( c_{\top} \) and \( c_{\bot} \) constitute the extremal elements of this partial order: \( c_{\bot} \) is an extension of every discourse context, and every discourse context is in turn an extension of \( c_{\top} \).

Fact 1 (Partial order and extrema).

- \( \geq \) forms a partial order on \( \mathcal{C} \)
- For every \( c \in \mathcal{C} \): \( c_{\bot} \geq c \geq c_{\top} \)

Finally, for any two discourse contexts \( c \) and \( c' \), we will refer to \( c \cap c' \) as the merge of \( c \) and \( c' \). It can be shown that \( c \cap c' \) is always a proper discourse context, i.e., a non-empty downward closed set of states, and moreover, that the information available/requested in \( c \cap c' \) is exactly the information available/requested in \( c \) plus the information available/requested in \( c' \).

Definition 12 (Merging two discourse contexts).

For any \( c, c' \in \mathcal{C} \), \( c \cap c' \) is called the merge of \( c \) and \( c' \).

Fact 2 (Merging yields a new discourse context).

For any \( c, c' \in \mathcal{C} \), \( c \cap c' \) is also in \( \mathcal{C} \).

Fact 3 (Merging information and issues). Let \( c, c' \in \mathcal{C} \). Then:

1. A possible world is compatible with the information available in \( c \cap c' \) just in case it is compatible with the information available in \( c \) and with the information available in \( c' \).
2. A state settles all the issues raised in \( c \cap c' \) just in case it settles all the issues raised in \( c \) and all the issues raised in \( c' \).

This completes our brief exploration of discourse contexts. We are now ready to turn to meanings.

4 Meanings and propositions

In section 2 we characterized the meaning of a sentence in general terms as its context change potential, which can be modeled formally as a function that maps every discourse context to a new discourse context. We have now specified what we take discourse contexts to be, so it has also become clearer what we take meanings to be.

However, we will not regard just any function \( f \) over discourse contexts as a meaning function. First of all, we will assume meanings to be functions that map any discourse context \( c \) to a new discourse context that is an extension of \( c \). Second, if we have two discourse contexts \( c \) and \( c' \), one an extension of the
other, we will assume a particular relation to hold between \( f(c) \) and \( f(c') \). The idea is that if \( f(c) \) and \( f(c') \) differ, this difference should be traceable to the initial difference in information and issues between \( c \) and \( c' \). Once the initial gap between \( c \) and \( c' \) is filled, the difference between \( f(c) \) and \( f(c') \) should also vanish.

Let us make this intuition precise. Consider a discourse context \( c \), and an extension of it, \( c' \). Now consider \( f(c') \) and \( f(c) \). First of all, we want it be the case that \( f(c') \) is still an extension of \( f(c) \). That is, \( f \) must be a monotonic function with respect to the extension order on discourse contexts. However, we will require something stronger than this: namely, if we add the information and the issues present in \( c' \) to \( f(c) \), i.e., if we take the merge of \( c' \) and \( f(c) \), we should end up exactly in \( f(c') \). We will refer to this condition as the compatibility condition.

**Definition 13** (Compatibility condition).
A function \( f \) over discourse contexts satisfies the compatibility condition if and only if for every \( c,c' \in \mathcal{C} \) such that \( c' \geq c \), we have that \( f(c') = f(c) \cap c' \).

**Definition 14** (Meanings).
A meaning is a function \( f \) which maps every discourse context \( c \) to a new discourse context \( f(c) \geq c \), in compliance with the compatibility condition.

Now recall that in the classical setting, the meaning of a sentence \( \varphi \) is identified with its truth-conditions—a function from worlds to truth-values—or equivalently, with the set of worlds \( |\varphi| \) that satisfy these truth-conditions. This set of worlds is usually referred to as the *proposition* expressed by the sentence. Furthermore, as pointed out above, in the classical setting the proposition expressed by a sentence \( \varphi \) is taken to determine the sentence’s context change potential: it is assumed that the intended effect of an utterance of \( \varphi \) is to restrict the discourse context—classically modeled as a set of possible worlds—to \( |\varphi| \). In other words, the new discourse context is obtained by intersecting the old discourse context with the proposition expressed by \( \varphi \).

In the present setting, we may also introduce a notion of propositions that fulfills precisely the same role. To get at the right notion, recall from fact 1 that every discourse context \( c \) is an extension of \( c_T \). Thus, the compatibility condition ensures that for every meaning \( f \) and every discourse context \( c \):

\[ f(c) = f(c_T) \cap c \]

This means that \( f \) is completely determined by \( f(c_T) \). If we have a certain discourse context \( c \) and we want to know what the new discourse context is that results from applying \( f \) to \( c \), we can simply take the intersection of \( c \) with \( f(c_T) \). Thus, just like in the classical case, a meaning \( f \) can be identified with a unique static object, \( f(c_T) \), which we will call the *proposition* associated with \( f \). So our new notion of meanings also gives rise to a new notion of propositions. Namely, propositions are non-empty, downward closed sets of states.
Definition 15 (Propositions).

- A proposition is a non-empty, downward closed set of states.
- The set of all propositions is denoted by $\Pi$.

We will proceed to characterize some special properties that propositions may have. Notice that there is a one-to-one correspondence between propositions and meanings: for every meaning $f$, the associated proposition is $f(c^\top)$, and for any proposition $A$, the associated meaning is the function $f_A$ that maps every discourse context $c$ to $c \cap A$. This means that everything we say below about propositions directly pertains to the associated meanings as well.

Also notice that, as in the classical setting, propositions and discourse contexts are the same type of objects. This means that many properties of discourse contexts that we discussed in section 3 can also be predicated, in much the same way, of propositions.

For instance, we will say that the informative content of a proposition $A$, $\text{info}(A)$, is the information that is available in the discourse context $c^\top \cap A$. Since $c^\top = \varphi(\omega)$, we have that $c^\top \cap A = A$. Thus, the information that is available in the discourse context $c^\top \cap A$ is embodied by $\bigcup A$.

Definition 16 (Informative content of a proposition).

- For any proposition $A$: $\text{info}(A) = \bigcup A$

We will say that a state $t$ settles a proposition $A$ just in case it settles the issue in $c^\top \cap A$, which is the case if and only if $t \in A$.

Definition 17 (Settling a proposition).

- An information state $t$ settles a proposition $A$ if and only if $t \in A$.

We will say that a proposition $A$ is informative just in case its informative content is non-trivial, i.e., $\text{info}(A) \neq \omega$. We will say that $A$ is inquisitive just in case it is not settled by its own informative content, i.e., $\text{info}(A) \not\in A$.

Definition 18 (Informative and inquisitive propositions).

- A proposition $A$ is informative iff $\text{info}(A) \neq \omega$.
- A proposition $A$ is inquisitive iff $\text{info}(A) \not\in A$.

These notions of informative and inquisitive propositions are closely related to the notions of informed and inquisitive discourse contexts defined above. Namely, a proposition $A$ is informative just in case the associated meaning $f_A$ can turn an ignorant discourse context into an informed discourse context, and similarly, a proposition is inquisitive just in case the associated meaning can turn an indifferent discourse context into an inquisitive discourse context.
Fact 4 (Informative and inquisitive propositions and discourse contexts).

- A proposition \( A \) is informative iff there is at least one discourse context \( c \) such that \( c \) is ignorant and \( f_A(c) \) is informed.

- A proposition \( A \) is inquisitive iff there is at least one discourse context \( c \) such that \( c \) is indifferent and \( f_A(c) \) is inquisitive.

Just like we identified two special discourse contexts, \( c_{\top} \) and \( c_{\bot} \), we can also identify two special propositions, namely the tautological proposition \( A_{\top} := \wp(\omega) \), whose associated meaning maps every discourse context to itself, and the contradictory proposition \( A_{\bot} := \{\emptyset\} \), whose associated meaning maps every discourse context to the absurd context.

Definition 19 (Tautology and contradiction).

- \( A_{\top} := \wp(\omega) \)

- \( A_{\bot} := \{\emptyset\} \)

Just like discourse contexts, propositions can be ordered either in terms of their informative component or in terms of their inquisitive component. It is natural to regard one proposition \( A \) as being at least as informative as another proposition \( B \) in case the meaning \( f_A \) provides at least as much information as \( f_B \), in the sense that for any context \( c \), \( f_A(c) \) is at least as informed as the context \( f_B(c) \). It is easy to see that this is the case if and only if \( \text{info}(A) \subseteq \text{info}(B) \).

Definition 20 (Informative order on propositions).
Let \( A, B \in \Pi \). Then:

- \( A \models_{\text{info}} B \) iff \( \text{info}(A) \subseteq \text{info}(B) \)

Similarly, if \( A \) and \( B \) are equally informative, then it is natural to regard \( A \) as being at least as inquisitive as \( B \) just in case for any context \( c \), \( f_A(c) \) is at least as inquisitive as \( f_B(c) \). This is the case if and only if \( A \subseteq B \).

Definition 21 (Inquisitive order on propositions).

Let \( A, B \in \Pi \) and \( \text{info}(A) = \text{info}(B) \). Then:

- \( A \models_{\text{inq}} B \) iff \( A \subseteq B \)

We will then say that one proposition \( A \) entails another proposition \( B \) just in case (i) \( A \) is at least as informative as \( B \), and (ii) \( A \) is at least as inquisitive as the proposition that results from restricting \( B \) to the informative content of \( A \), i.e., \( A \models_{\text{inq}} B \mid \text{info}(A) \), where \( B \mid \text{info}(A) := \{s \cap \text{info}(A) \mid s \in B\} \). It can be shown that these two conditions are satisfied exactly if \( A \subseteq B \). So entailment can simply be defined in terms of inclusion.

Definition 22 (Entailment).

Let \( A, B \in \Pi \). Then:

- \( A \models B \) iff \( A \subseteq B \)
The entailment order on propositions is closely related to the extension order on discourse contexts. Namely, $A$ entails $B$ just in case for any discourse context $c$, $f_A(c)$ is an extension of $f_B(c)$.

**Fact 5** (Entailment and extension). Let $A, B \in \Pi$. Then:

- $A \models B$ iff for any $c \in \mathcal{C}$, $f_A(c) \geq f_B(c)$

Entailment forms a partial order on $\Pi$, and $A_\top$ and $A_\bot$ constitute the extremal elements of this partial order: $A_\bot$ entails every proposition, and every proposition in turn entails $A_\top$.

**Fact 6** (Partial order and extrema).

- $\models$ forms a partial order on $\Pi$
- For every $A \in \Pi$: $A_\bot \models A \models A_\top$

In fact, it can be shown that $\langle \Pi, \models \rangle$ forms a complete Heyting algebra, with infinitary meet and join operators, and a relative pseudo-complement operator (Roelofsen, 2011). Incidentally, the meet of a set of propositions amounts to their intersection, and the join of a set of propositions amounts to their union, just as in the complete Boolean powerset algebra that underlies classical logic.

This completes our presentation of the basic inquisitive semantics framework (IS for short). This basic framework may be further generalized in several directions. For instance, we may consider partial meaning functions, which are only defined for certain input contexts; admitting this kind of meanings allows for a natural account of presuppositions, as discussed in Ciardelli et al. (2012). Second, we may consider meaning functions that satisfy a weaker version of the compatibility condition. Such meanings would no longer be fully determined by a unique, static proposition; they would be genuinely dynamic. Finally, in order to model more than just informative and inquisitive content we may further enrich our notion of discourse contexts, adding additional structure, and thereby also obtain an even richer notion of meaning (see, e.g., Ciardelli et al., 2009, 2013b).

For now, however, we will restrict our attention to the basic IS framework. In the next section we will illustrate how this framework can be used to capture the informative and inquisitive content of various types of sentences in natural language. After that, in section 6, we will situate the framework in the context of previous work on inquisitive aspects of meaning.

## 5 Illustration

The meaning of a wide range of declarative and interrogative sentence types can be captured in a uniform way in the IS framework presented here. To illustrate this, consider the following examples, where ↑ indicates rising intonation:
Figure 1: Propositions expressed by (1)–(5), exemplifying several types of declarative and interrogative sentences (polar, disjunctive, conditional, wh-).

(1) Peter will attend the meeting.
(2) Will Peter attend the meeting?
(3) Will Peter attend the meeting, or Maria?
(4) If Peter attends the meeting, will Maria attend it too?
(5) Who will attend the meeting?

These sentences instantiate five different sentence types: (1) a simple declarative, (2) a polar interrogative, (3) an open disjunctive interrogative, (4) a conditional interrogative, and (5) a wh-interrogative.

The informative and inquisitive content of all these sentence types can be captured in a uniform way in $\text{IS}$. To see this, consider the propositions depicted in figure 1. We assume here for simplicity that our logical space consists of just four possible worlds: one world where both Peter and Maria will attend the meeting, one where only Peter will attend, one where only Maria will attend, and one where neither Peter nor Maria will attend—in figure 1, these worlds are labeled 11, 10, 01, and 00, respectively. The shaded areas in the figure represent the maximal elements of each proposition (recall that propositions are downward closed). We will refer to these maximal elements as the alternatives that each proposition consists of.\footnote{Our use of the term \textit{alternatives} here is very closely related to its use in the framework of \textit{alternative semantics} (Kratzer and Shimoyama, 2002; Simons, 2005; Alonso-Ovalle, 2006; Alonso-Ovalle, 2006; Aloni, 2007, among others). The connection between inquisitive semantics and alternative semantics is discussed in detail in Roelofsen (2012).}

The declarative sentence in (1) expresses the proposition depicted in figure 1(a). This proposition consists of a single alternative, which is the set of all worlds where Peter will attend the meeting. Notice that $\text{info}([\{(1)\}] \neq \omega$, which...
means that \( \text{info}(\{1\}) \) is informative. However, \( \text{info}(\{1\}) \in \{1\} \), which means that \( \{1\} \) is not inquisitive. Thus, the semantics captures that in uttering (1), a speaker provides the information that Peter will attend the meeting, and does not request any further information.

The polar interrogative in (2) expresses the proposition depicted in figure 1(b). This proposition consists of two alternatives: the set of all worlds where Peter will attend the meeting, and the set of all worlds where Peter will not attend the meeting. \( \text{info}(\{2\}) = \omega \), which means that \( \{2\} \) is not informative. However, \( \text{info}(\{2\}) \notin \{2\} \), which means that \( \{2\} \) is inquisitive. Thus, the semantics captures that in uttering (2), a speaker does not provide any information, but requests enough information from other participants to determine whether Peter will attend the meeting or not. This example illustrates the general fact that if a proposition contains at least two alternatives, then it is always inquisitive. After all, if a proposition \( A \) is not inquisitive, then \( \text{info}(A) \in A \), which means that \( \text{info}(A) \) must be the unique alternative for \( A \).

The open disjunctive interrogative in (3) expresses the proposition depicted in figure 1(c). This proposition consists of three alternatives: the set of all worlds where Peter will attend the meeting, the set of all worlds where Maria will attend, and the set of all worlds where neither Peter nor Maria will attend. Since \( \text{info}(\{3\}) = \omega \), \( \{3\} \) is not informative. However, having three distinct alternatives, \( \{3\} \) is inquisitive. Thus, in uttering (3), a speaker does not provide any information, but requests enough information from other participants to determine whether Peter will attend the meeting, or Maria, or none of the two.

The conditional interrogative in (4) expresses the proposition depicted in figure 1(d). This proposition consists of two alternatives, which correspond to the following answers to (4):

(6)   a. Yes, if Peter attends the meeting, then Maria will attend as well.
     b. No, if Peter attends the meeting, then Maria won’t attend.

Notice that \( \text{info}(\{4\}) = \omega \), which means that \( \{4\} \) is not informative. However, having two distinct alternatives, \( \{4\} \) is inquisitive. Thus, in uttering (4), a

\[\text{The converse does not hold: if a proposition is inquisitive, it does not necessarily have two or more alternatives. As an instance, consider a proposition that contains an infinite chain of states, } s_1 \subset s_2 \subset s_3 \subset \ldots \text{, without any maximal element. By definition 18, such propositions are inquisitive, even though they do not contain any alternatives. So, if a proposition contains at least two alternatives, then it is inquisitive, but the reverse implication does not hold in general (Ciardelli, 2009, 2010).}\]

\[\text{Intuitively, there are other natural answers to this conditional interrogative as well:}\]

(i)   a. No, it might be that Peter will attend the meeting, but Maria won’t.
     b. Peter won’t attend the meeting.

The first assumes a modal interpretation of (4), the second does not resolve the conditional interrogative as intended but rather denies its antecedent. The basic IS framework is not fine-grained enough to capture the special nature of these answers. However, it may be further extended in such a way that a more refined semantic analysis of conditional interrogatives becomes possible, capturing both modal interpretations and denials of the antecedent (Groenendijk and Roelofsen, 2010).
speaker does not provide any information, but requests enough information from other participants to determine whether Maria will attend the meeting, under the assumption that Peter will.

Finally, consider the wh-interrogative in (5). In uttering this sentence, a speaker may be taken to request a complete, exhaustive specification of all people who will attend the meeting. Alternatively, she may be taken to request enough information to identify at least one person who will attend the meeting, if there is such a person, and otherwise to establish that nobody will. These two interpretations of wh-interrogatives are generally referred to as mention-all and mention-some interpretations, respectively. Under a mention-all interpretation, (5) expresses the proposition depicted in 1(e). Under a mention-some interpretation, it expresses the proposition depicted in 1(c). In both cases, the proposition expressed is inquisitive but not informative. But under the mention-all interpretation the proposition expressed is more inquisitive than under the mention-some interpretation.

This completes our (non-exhaustive) illustration of the range of sentence types that can be treated in IS. Along the way, we also drew attention to several aspects of meaning that are beyond the scope of this basic inquisitive framework, and we pointed to places in the literature where the framework is further extended to overcome these limitations.

To end this section, we would like to emphasize that we take the meaning of a sentence to determine the intended effect of an utterance of that sentence on the discourse context. Whether this effect is achieved depends on how other discourse participants react to the utterance. Thus, an utterance can be thought of as a proposal to accept a certain piece of information and to settle a certain issue. Other participants may comply with such a proposal, but they may also reject it, or come up with a counter-proposal. For an analysis of this interactive process, see Groenendijk (2008); Balogh (2009); Farkas and Roelofsen (2012).

We end the paper by relating the ideas presented here to previous work on inquisitive notions of meaning.

6 Previous inquisitive notions of meaning

There is a large body of work on the semantics of questions, which has given rise to several inquisitive notions of meaning. We will restrict our attention here to those proposals that are most closely related to our own. That is, we will consider the classical work on the semantics of questions by Hamblin (1973), Karttunen (1977), and Groenendijk and Stokhof (1984), and a number of more recent theories that are couched in a dynamic semantic framework (Jäger, 8). In uttering a wh-interrogatives like (5), a speaker is often taken to presuppose that at least one person will attend the meeting. Such a presupposition cannot be captured directly in the basic IS framework, which is only designed to capture at-issue informative and inquisitive content (see also footnote 4 above). However, if we allow for partial meaning functions, i.e., meaning functions that are only defined for certain input contexts, then presuppositions can be captured in a natural way as well (see Ciardelli et al., 2012).
Questions as sets of answers. The central idea in Hamblin (1973) is that “questions set up a choice-situation between a set of propositions, namely those propositions that count as answers to it” (Hamblin, 1973, p.48). Thus, Hamblin takes questions denote sets of classical propositions.

In the closely related theory of Karttunen (1977), questions also denote sets of classical propositions, but only those propositions that correspond to true answers. Thus, in both systems, the meaning of a question is a function from worlds to sets of classical propositions. In Hamblin’s system, this function maps every possible world to the same set of propositions, corresponding to the set of all possible answers; in Karttunen’s system, every world is mapped to a subset of all possible answers, namely those that are true in the given world. As noted by Karttunen (1977, p.10), the difference is not essential. In both cases, the meaning of a question is fully determined by—and could be identified with—the set of all classical propositions that correspond to a possible answer.

A fundamental problem with these accounts is that they do not specify in more detail what “possible answers” are supposed to be. Of course, Hamblin and Karttunen do provide a compositional semantics for a fragment of English, and thereby specify what they take to be the possible answers to the questions in that fragment. But in order for these theories to be evaluated, we first need to know what the notion of a “possible answer” is supposed to capture. To illustrate this point, consider the following example:

(7) Who is coming for dinner tonight?
   a. Paul is coming.
   b. Only Paul and Nina are coming.
   c. Some girls from my class are coming.
   d. I don’t know.

In principle, all the responses in (7a-d) could be seen as possible answers to (7). For Hamblin and Karttunen, only (7a) counts as such. However, it is not made clear what the precise criteria are for being considered a possible answer, and on which grounds (7a) is to be distinguished from (7b-d).

In IS, question-meanings are also sets of classical propositions, just like in Hamblin’s and Karttunen’s approach. However, in IS these classical propositions are not thought of as the “possible answers” to the question. Rather, they are thought of as the information states that settle the issue that the question raises, or equivalently, as the pieces of information that resolve the issue that the question raises. As a consequence, in IS question-meanings cannot be defined as arbitrary sets of classical propositions, which is what Hamblin and Karttunen take them to be. Rather, they should be defined as downward closed sets of classical propositions. After all, if \( \alpha \) is a piece of information that resolves the issue raised by a given question \( Q \), then any stronger piece of information \( \beta \subset \alpha \) will also resolve the issue raised by \( Q \).
In our view, intuitions about which pieces of information resolve a given issue are much more robust than intuitions about what the “possible answers” to a given question are. Thus, evaluating theories of questions formulated in IS is more straightforward than evaluating theories of questions which, like Hamblin and Karttunen’s, identify question-meanings with sets of possible answers.

This said, we should hasten to emphasize two things. First, IS does not in itself constitute a complete theory of questions. Rather, it constitutes a semantic framework in which such theories may be formulated. It provides a well-circumscribed notion of meaning, but it does not specify how such meanings should be assigned to sentences in natural language.

Second, even though there is not supposed to be a one-to-one correspondence between the classical propositions that constitute a question-meaning in IS and the “possible answers” to that question, this is not to say that question-meanings in IS cannot play a role in characterizing sensible answerwork relations at all. Indeed, one natural way to characterize the basic semantic answers to a question Q is as those pieces of information that (i) resolve the issue raised by Q and (ii) do not provide more information than necessary to do so, i.e., are not strictly stronger than any other piece of information that also resolves Q. Hence, the basic semantic answers to Q may be taken to coincide with the alternatives in the proposition expressed by Q.9,10 Along the same lines, we may also define notions of partial answerhood and subquestionhood (see, e.g., Groenendijk and Roelofsen, 2009), which are crucial for the analysis of discourse and information structure (see, e.g., Roberts, 1996; Büring, 2003; Beaver and Clark, 2008).

Now let us return to the theories of Hamblin and Karttunen. Besides the issue just discussed, there is another problem with the notion of question-meanings that these theories adopt. As pointed out by Groenendijk and Stokhof (1984), it is difficult to define a suitable notion of entailment that determines when one of these meanings is stronger than another. One consequence of this is that it is hard, if not impossible, to give a principled account of the interaction of questions with logical connectives and quantifiers. It proves problematic, for instance, to give a satisfactory definition of conjunction of questions.

This problem does not arise in IS. In this framework, there is a well-behaved notion of entailment, which compares propositions in terms of their informative and inquisitive content. As mentioned above, the space of propositions in IS, ordered by entailment, has a certain algebraic structure, and a natural treatment

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9Recall from footnote 6 that some propositions in $\text{In}_B$ do not contain any alternatives. For a discussion of the notion of answerhood in the case of these propositions, see Ciardelli (2010) and Ciardelli et al. (2013b).

10Notice that the approach to answerwork sketched here constrains the sets of basic answers to a question in a particular way. Call a set $S$ of states a set of alternatives if for any two distinct states $s, t \in S$, neither $s \subseteq t$ nor $t \subseteq s$. The set of alternatives in a given proposition $A$ is always a set of alternatives in this technical sense, and vice versa, any set of alternatives is the set of alternatives of some proposition. Our definition of basic semantic answers to a question as minimal resolving pieces of information implies that the set of basic semantic answers has to be a set of alternatives. Thus, even if—in the spirit of Hamblin and Karttunen—we were to identify the meaning of a question with the set of its basic semantic answers, not just any set of classical propositions would count as a proper question meaning.
of the logical constants is obtained by associating them with the basic operations in this space. Thus, for instance, the classical treatment of conjunction as a meet operation can be generalized in IS to apply to informative and inquisitive sentences in a uniform way, and the same goes for the other operations (see Roelofsen, 2011; Ciardelli et al., 2012).

Questions as partitions. According to Groenendijk and Stokhof (1984), a question denotes, in each world, a single classical proposition embodying the true exhaustive answer to the question in that world. For instance, if \( w \) is a world in which Paul and Nina are coming for dinner, and nobody else is coming, then the denotation of (7) in \( w \) is the classical proposition expressed by (7b).

The meaning of a question, then, is a function from worlds to classical propositions. These classical propositions have two special properties: they are mutually exclusive (since two different exhaustive answers are always incompatible), and together they form a cover of the entire logical space (since every world is compatible with at least one exhaustive answer). So the meaning of a question can be identified with a set of classical propositions which together form a partition of logical space.

Partitions correspond to a specific kind of propositions in IS. That is, for every partition \( P \), there is a corresponding proposition \( A_P \) in IS, consisting of all states that are contained in one of the blocks in \( P \): \( A_P := \{ s \subseteq b \mid b \in P \} \). On the other hand, not every proposition in IS corresponds to a partition. This holds in particular for all propositions in IS that contain overlapping alternatives, and ones whose elements do not cover the entire logical space.

Thus, the notion of meaning presented here is more general than the notion of meaning in partition semantics. This generalization is needed to capture the meaning of certain types of questions in natural language. In particular, in a partition semantics it is difficult, if not impossible, to give a satisfactory account of conditional questions, disjunctive questions, and mention-some wh-questions, exemplified in (3)–(5) above—as we saw, these types of questions all express propositions with overlapping alternatives. Thus, in terms of empirical coverage, inquisitive semantics has a significant advantage over partition semantics.

Questions in dynamic semantics. There are a number of theories that aim to capture the semantics of questions in a dynamic framework. The first such theories, developed by Jäger (1996), Hulstijn (1997), and Groenendijk (1999), essentially reformulate the partition theory of questions in the format of an update semantics (Veltman, 1996).\(^ {11} \) This means that they explicitly identify meanings with context change potentials, i.e., functions over discourse contexts, just as we did in the present paper. Moreover, rather than modeling a discourse context simply as a set of worlds—embodying the information established so far—these theories provide a more refined model of the discourse context, one that also embodies the issues that have been raised so far. More specifically, a

\(^ {11} \text{See the book Questions in dynamic semantics (Aloni et al., 2007) for several papers elaborating on these early proposals.} \)
discourse context is modeled as an equivalence relation $R$ over a set of worlds $s \subseteq \omega$. Such an equivalence relation can be taken to encode both information and issues. On the one hand, the set $s$, which is called the domain of $R$, can be taken to consist precisely of those worlds that are compatible with the information established so far. And on the other hand, $R$ can be taken to relate two distinct worlds $w$ and $v$ in $s$ just in case the difference between $w$ and $v$ is not (yet) at-issue, i.e., the discourse participants have not yet expressed an interest in information that would distinguish between $w$ and $v$. In other words, $R$ can be conceived of as a relation encoding indifference (Hulstijn, 1997).

Both assertions and questions can then be taken to have the potential to change the context in which they are uttered. Assertions restrict the domain $s$ to those worlds in which the asserted sentence is true (strictly speaking, they remove all pairs of worlds $\langle w, v \rangle$ from $R$ such that the asserted sentence is false in at least one of the two worlds). Questions disconnect worlds, i.e., they remove a pair $\langle w, v \rangle$ from $R$ just in case the true exhaustive answer to the question in $w$ differs from the true exhaustive answer to the question in $v$.

Thus, the dynamic systems of Jäger (1996), Hulstijn (1997), and Groenendijk (1999) provide a notion of context and meaning that embodies both informative and inquisitive content in a uniform way. However, as discussed in detail by Mascarenhas (2009), several issues, both empirical and conceptual, remain. Empirically, it is difficult, if not impossible, to deal with conditional questions, disjunctive questions, and mention-some constituent questions in this framework.\footnote{Although see Isaacs and Rawlins (2008) for an analysis of conditional questions in a dynamic partition semantics that allows for hypothetical updates of the context of evaluation.} Clearly, these problems are inherited from the classical partition theory of questions (see above).

Conceptually, if $R$ is primarily thought of as a relation encoding indifference, then it is not clear why it should always be an equivalence relation. In particular, it is not clear why $R$ should always be transitive. The discourse participants could very well be interested in information that distinguishes $w$ from $v$, while they are not interested in information that distinguishes either $w$ or $v$ from a third world $u$. To model such a situation, we would need an indifference relation $R$ such that $\langle w, u \rangle \in R$ and $\langle u, v \rangle \in R$ but $\langle w, v \rangle \notin R$. This is impossible if we require $R$ to be transitive.

These concerns led Groenendijk (2009) and Mascarenhas (2009) to develop a system in which indifference relations are defined as reflexive and symmetric, but not necessarily transitive relations. Otherwise, the architecture of this system is still essentially the same as that of Jäger (1996), Hulstijn (1997), and Groenendijk (1999). In particular, discourse contexts are still defined as sets of world-pairs. Groenendijk (2009) and Mascarenhas (2009) argue that their semantics, besides addressing the conceptual issue concerning indifference relations, also overcomes the empirical issues concerning conditional questions, disjunctive questions, and mention-some constituent questions. However, whereas open disjunctive questions with two disjuncts, like (3) above, can be dealt with satisfactorily, disjunctive questions with three or more disjuncts are still prob-
lematic, and the same holds for mention-some constituent questions. Unfortunately, we do not have space here to show exactly why these problems arise; a detailed explanation can be found in Ciardelli and Roelofsen (2011).

This observation has led to the development of the IS framework presented here. Several important aspects of the general philosophy underlying the framework developed by Groenendijk (2009) and Mascarenhas (2009) persist in the present framework. However, its key ingredient—the notion of meaning—has been generalized, and this generality is needed to suitably capture the meaning of open disjunctive questions and mention-some constituent questions. Thus, the IS framework presented here naturally fits within the tradition of dynamic semantic theories of informative and inquisitive discourse, but it is more general and empirically more adequate than its predecessors.

References


