Issues in epistemic change

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Introduction

Goal of the talk

Develop a simple formal framework to model and reason about:

- The beliefs that an agent has
- The issues that she entertains
- How these change in the process of inquiry

Bringing together ideas from

- Interrogative belief revision (Olsson & Westlund'06, Enqvist'09)
- Dynamic epistemic logic (vDitmarsch et.al.'07, vBenthem'13)
- Inquisitive semantics (Ciardelli, Groenendijk & Roelofsen'13)

Overview

- Motivation (Olsson & Westlund'06)
- Building the framework
  - Knowledge and beliefs (vBenthem'07, vDitmarsch'05, Baltag & Smets'06)
  - Knowledge and issues (Ciardelli & Roelofsen'14)
  - Knowledge, beliefs, and issues
  - Dynamics
- Some applications
  - pertinent beliefs
  - research agenda
  - inquisitive contraction

Motivation

- Traditional theories of epistemic change construe the epistemic state of an agent simply as a set of beliefs
- Olsson & Westlund (2006) argue that this does not give a full picture of the process of epistemic change
- Besides the beliefs of an agent, we also have to take her epistemic goals into account
- These epistemic goals amount to the issues that she entertains
- Olsson & Westlund refer to this set of issues as the agent's research agenda

Motivation

There are interesting and systematic connections between changes in the beliefs of an agent and her research agenda

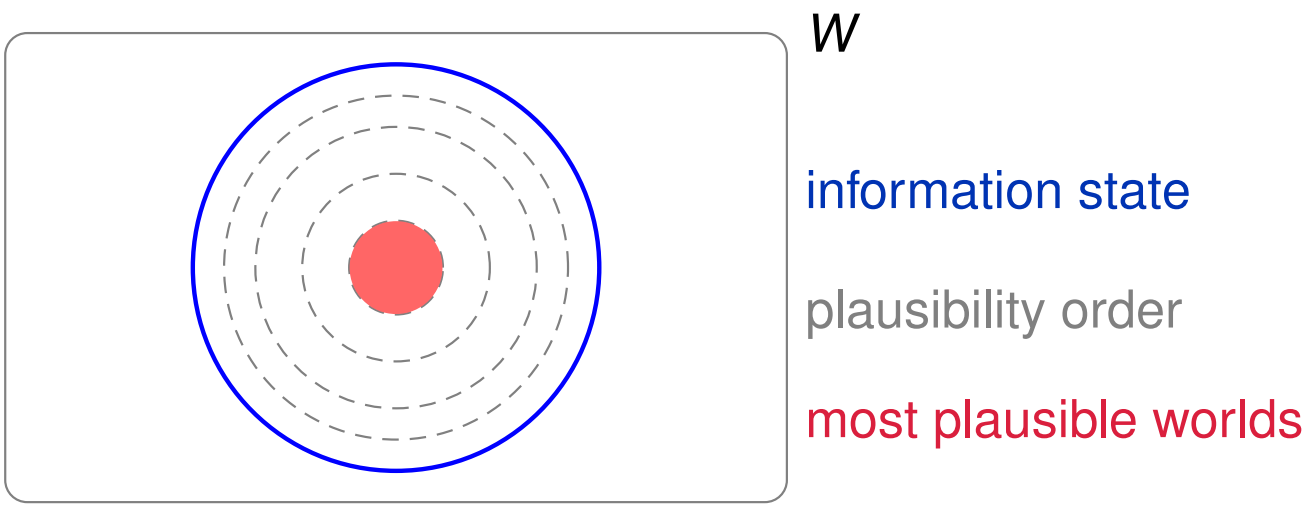
Example

(Enqvist '10)

- A scientist investigates the anomalous orbit of a planet.
- Two promising competing hypotheses: $H1$ and $H2$.
  1. Belief: $H1 \vee H2$
  2. Question on the research agenda: $\{H1, H2\}$
- Attempts to verify either hypothesis fail.
- As a result, a third hypothesis is considered.
- This affects both the beliefs and the research agenda:
  1. New belief: $H1 \vee H2 \vee H3$
  2. New question on the research agenda: $\{H1, H2, H3\}$

Knowledge and Beliefs

- An information state is a set of possible worlds.
- A plausibility order over an information state s is a well-preorder of s, that is, a relation $\leq$ satisfying:
  - reflexivity: for any $w \in s$, $w \leq w$;
  - transitivity: for any $w, v, u \in s$, if $w \leq v$ and $v \leq u$ then $w \leq u$;
  - every non-empty set of worlds has a $\leq$-minimal element.



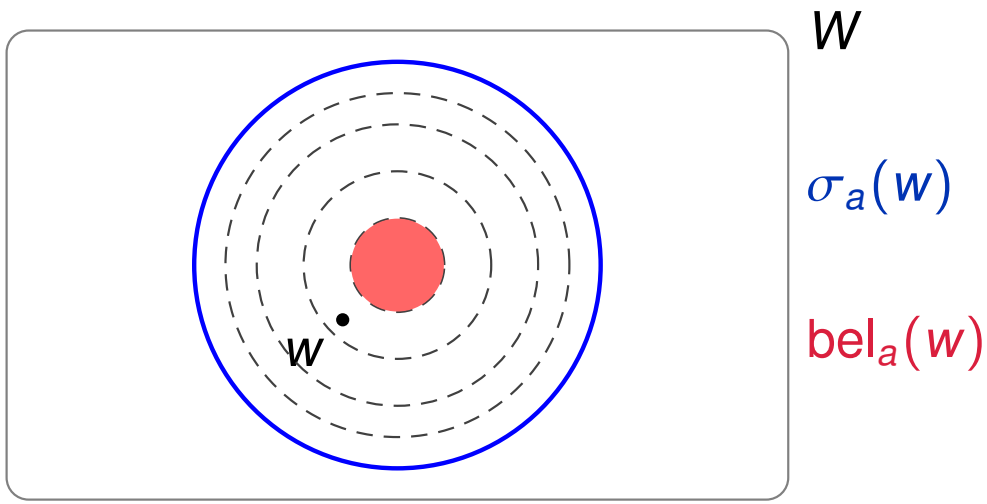
Knowledge and Beliefs

Epistemic plausibility models

An epistemic plausibility model for a set $\mathcal{A}$ of agents consists of:

- a set W of possible worlds
- a valuation function V:
  for every $w \in W$, $V(w)$ is a set of atomic sentences
- an epistemic map σ_a for each agent $a \in \mathcal{A}$:
  for every $w \in W$, $\sigma_a(w)$ is an information state
- a plausibility map $\leq_a$ for each agent $a \in \mathcal{A}$:
  for every $w \in W$, $\leq_a^w$ is a plausibility order over $\sigma_a(w)$

Knowledge and Beliefs



Modalities

- $M, w \models K_a\varphi \iff \forall v \in \sigma_a(w),\ M, v \models \varphi$
- $M, w \models B_a\varphi \iff \forall v \in \mathrm{bel}_a(w),\ M, v \models \varphi$

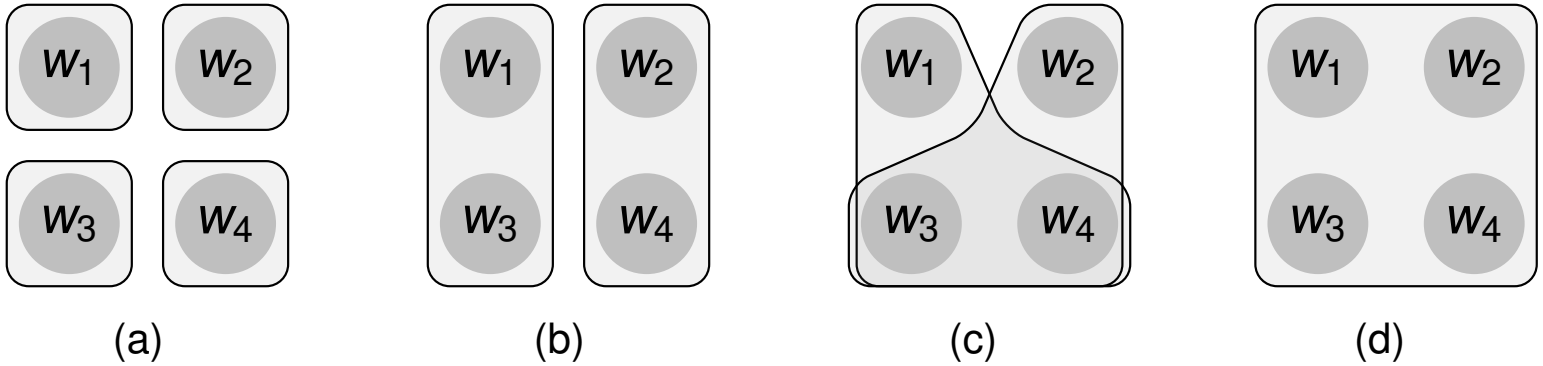
Knowledge and Issues

Issues

An issue over a state s is a non-empty, downward-closed cover of s.

Examples

Some issues over $\{w1, w2, w3, w4\}$, in decreasing order of strength.



Note: only maximal elements are displayed.

Inquisitive epistemic logic (IEL)

Inquisitive epistemic models

An inquisitive epistemic model for a set $\mathcal{A}$ of agents consists of:

- a set W of possible worlds
- a valuation function V:
  for every $w \in W$, $V(w)$ is a set of atomic sentences
- an epistemic map σ_a for each agent $a \in \mathcal{A}$:
  for every $w \in W$, $\sigma_a(w)$ is an information state
- an inquisitive map Σ_a for each agent $a \in \mathcal{A}$:
  for every $w \in W$, $\Sigma_a(w)$ is an issue over $\sigma_a(w)$

Inquisitive epistemic logic (IEL)

Language of IEL

(simplified fragment of C&R'14)

To talk about issues, we enrich the standard language of EL with interrogatives and with modalities that can embed interrogatives.

Declaratives	α	$::=$	$p \mid \neg\alpha \mid \alpha \wedge \alpha \mid K_a\alpha \mid K_a\mu \mid E_a\mu$
Interrogatives	μ	$::=$	$?\{\alpha, \ldots, \alpha\}$

Abbreviation

$?\alpha := ?\{\alpha, \neg\alpha\}$

Example

$E_a ?K_b ?p$

Knowledge and Issues

Semantics

- Usually, a semantics specifies truth-conditions wrt worlds.
- For interrogatives, however, this does not seem suitable.
- Rather, we give resolution conditions wrt to information states.
- The resolution conditions of an interrogative $?\{\alpha_1, \ldots, \alpha_n\}$ depend on the truth conditions for $\alpha_1, \ldots, \alpha_n$.
- Viceversa, the truth conditions of declaratives $K_a\mu$ and $E_a\mu$ depend on the resolution conditions of the complement μ.
- So, truth and resolutions are defined by simultaneous recursion.

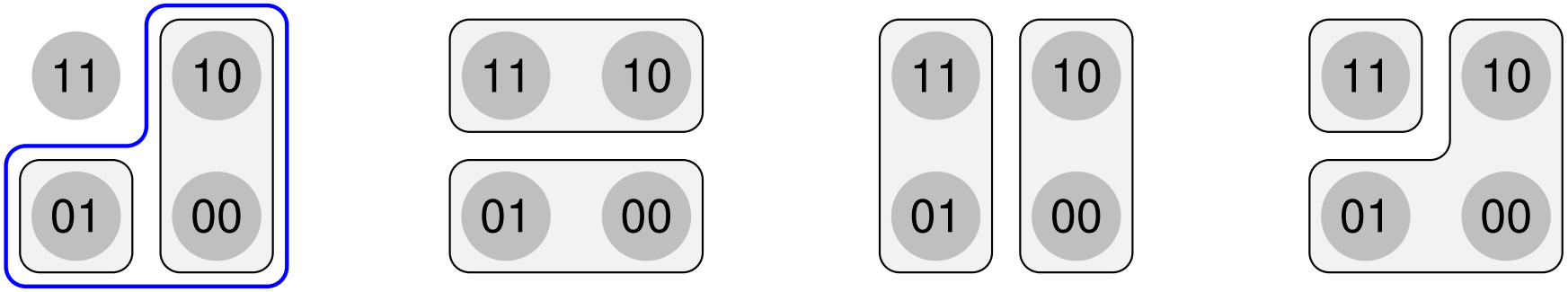
Knowledge and Issues

Resolution

- $M, s \models ?\{\alpha_1, \ldots, \alpha_n\} \iff$ for some α_i, $M, w \models \alpha_i$ for every $w \in s$

Truth

- $M, w \models K_a\mu \iff M, \sigma_a(w) \models \mu$
- $M, w \models E_a\mu \iff \forall t \in \Sigma_a(w),\ M, t \models \mu$
- All the remaining clauses are as usual.



$\sigma_a(w)$, $\Sigma_a(w)$ $\quad$ $?p$ $\quad$ $?q$ $\quad$ $?(p \wedge q)$

Interrogatives in minimal form

- We say that an interrogative $?\{\alpha_1, \ldots, \alpha_n\}$ is in minimal form in case for any equivalent interrogative $?\{\beta_1, \ldots, \beta_m\}$, it holds that $n \leq m$.

- For simplicity, we will assume throughout the talk that interrogatives are in minimal form.

Knowledge, Beliefs, and Issues

Inquisitive plausibility models

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- an plausibility map $\leq_a$ for each agent $a \in \mathcal{A}$
- an inquisitive map Σ_a for each agent $a \in \mathcal{A}$

Inquisitive belief logic (IBL)

Language of IBL

Declaratives $\alpha ::= p \mid \neg\alpha \mid \alpha \wedge \alpha \mid K\alpha \mid K\mu \mid E\mu \mid B\alpha \mid B\mu \mid E^B\mu$

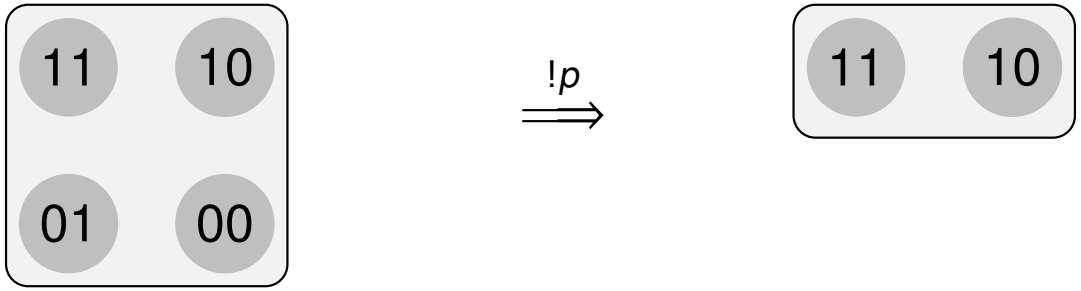
Interrogatives $\mu ::= ?\{\alpha, \ldots, \alpha\}$

Semantics of IBL

- $M, w \models B_a\mu \iff M, \mathrm{bel}_a(w) \models \mu$
- $M, w \models E_a^B\mu \iff \forall t \subseteq \mathrm{bel}_a(w)$ such that $t \in \Sigma_a(w) : \; M, t \models \mu$
- The other clauses are as above

Dynamics

- Epistemic actions are modeled in DEL as model transformations.
- Basic DEL deals with actions that affect knowledge.



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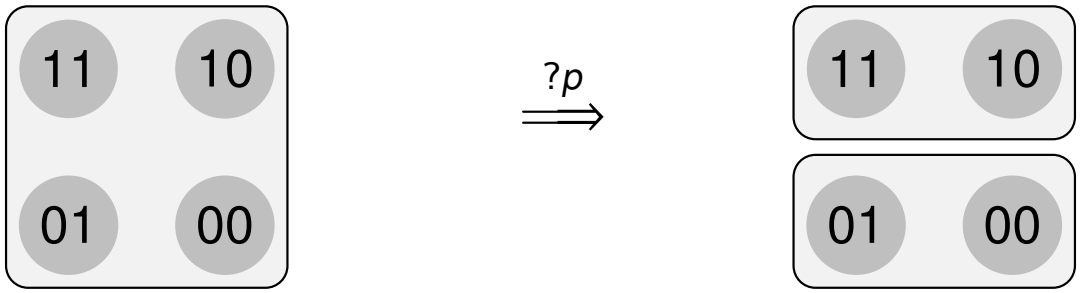
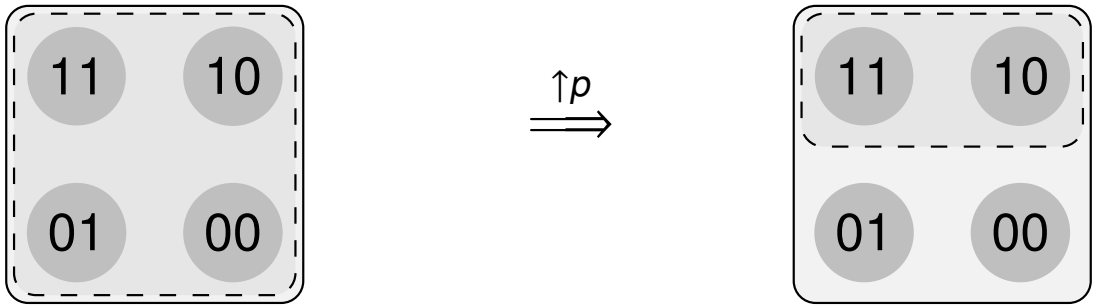
- Inquisitive DEL deals with actions that affect knowledge and issues.



Dynamics

- Epistemic actions are modeled in DEL as model transformation.
- Basic DEL deals with actions that affect knowledge.
- Inquisitive DEL deals with actions that affect knowledge and issues.
- Doxastic DEL deals with actions that affect knowledge and beliefs.



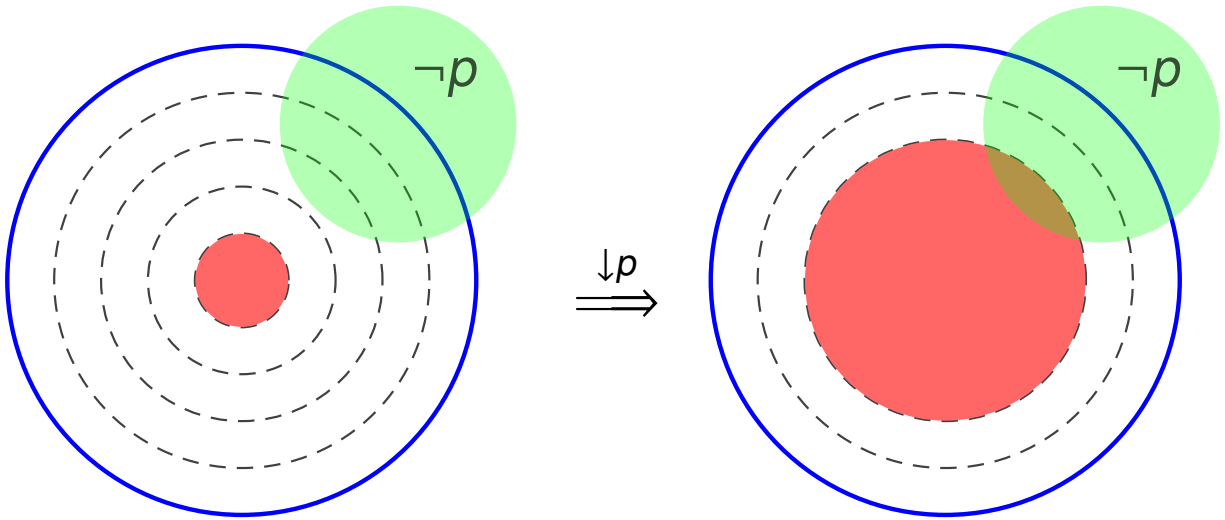
Revision and contraction

- Standard doxastic DEL approaches focus on revision, that is, on the action of adopting a new belief.
- In the process of adopting a new belief, some old belief may be given up. (contraction)
- However, we saw that contraction of a belief need not be followed by adoption of the opposite belief.
- Moreover, we saw that contraction may induce interesting changes in the research agenda.
- Therefore, we want to model contraction as a primitive action.

Possible recipes for contraction

Different recipes for contraction may be considered (just like for revision).

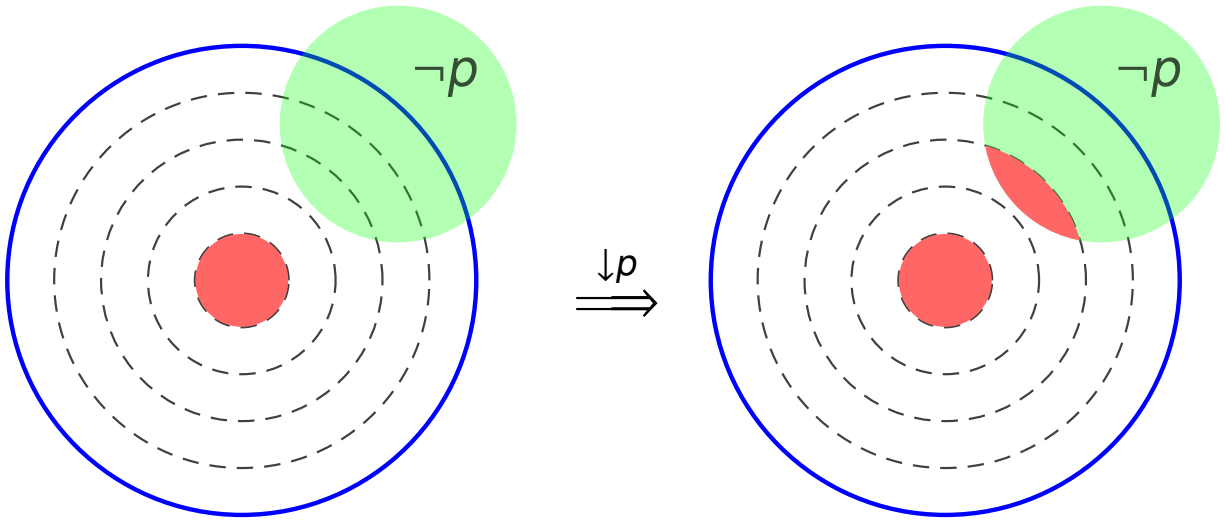
Example 1



Possible recipes for contraction

Different recipes for contraction may be considered (just like for revision).

Example 2



- We will not choose a specific recipe here.

- Rather, we will assume an arbitrarily chosen contraction operation, and show how it can be used to characterize interesting notions.

- We also add a corresponding dynamic modality to our language:

$$M, w \models [\downarrow \alpha]\beta \iff M^{\downarrow\alpha}, w \models \beta$$

Application 1: pertinent beliefs

Olsson and Westlund on pertinent beliefs (O&W'06, p.172)

"[An] adequate model should keep track not only of questions in need of answers but also of beliefs that answer questions. The latter have a special status. It is natural to think of them as having a higher degree of informational value than other beliefs. [...] The special status of question-answering beliefs should arguably be reflected in a formal model."

Application 1: pertinent beliefs

O&W's characterization

- O&W represent questions on the agenda syntactically, as sets of sentences $\{\alpha_1, \ldots, \alpha_n\}$, where $\alpha_1, \ldots, \alpha_n$ are the answers.
- They propose that a belief β qualifies as pertinent iff it is an answer to one of the questions on the agenda.

A problem

- Since questions are represented syntactically, β may be a pertinent belief and β' not, even though $\beta \equiv \beta'$.
- This problem does not arise in IBL, since questions are represented semantically.

Application 1: pertinent beliefs

Another problem

Intuitively, a belief is pertinent to μ not only if it *is* an answer to μ, but also if it constitutes an essential ground for believing an answer.

Example

- One of the goals of the agent is to establish $?p$.
- The agent believes q and $q \to p$.
- On these grounds, the agent believes p.
- In this case, both q and $q \to p$ should qualify as pertinent beliefs.
- They play an indispensable role in resolving the question $?p$.

Application 1: pertinent beliefs

Preliminary formal characterization

A belief β of an agent at M, w is pertinent in case there is a question $\mu = ?\{\alpha_1, \ldots, \alpha_n\}$ such that:

1. $M, w \models E\mu$
2. for some α_i: $\quad M, w \models B\alpha_i \ \wedge \ [\downarrow \beta]\neg B\alpha_i$

N.B.: It is important here that μ is assumed to be in minimal form

Still too restrictive

Intuitively, a belief is pertinent not only if it contributes to a complete answer, but also if it contributes to a partial answer.

Application 1: pertinent beleifs

Refined formal characterization

A belief β of an agent at M, w is pertinent in case there is a question $\mu = ?\{\alpha_1, \ldots, \alpha_n\}$ such that:

1. $M, w \models E\mu$
2. for some $\Gamma \subset \{\alpha_1, \ldots, \alpha_n\}$:

$$M, w \models B(\bigvee \Gamma) \wedge [\downarrow\beta]\neg B(\bigvee \Gamma)$$

Crucial points

To arrive at a fine-grained characterization of pertinent beliefs we crucially exploited two features of the $\mathrm{IEL}_{\mathrm{dox}}$ framework:

1. A semantic representation of questions.
2. The availability of a contraction operation.

Application 2: research agenda

- O&W argue that theories of epistemic change should take into account the agent's research agenda.
- The latter consists of the questions that the agent is aiming to resolve at the given stage of the inquiry.
- Enqvist (2010) argues that the agent's current agenda should not be seen as a primitive component of her epistemic state.
- Instead, it should be seen as determined by the agent's long term epistemic goals, and by her current beliefs.

- For us, the long term epistemic goals of an agent are encoded by the issues that the agent entertains.

- We say that a question $\mu = ?\{\alpha_1, \ldots, \alpha_n\}$ is on the research agenda of an agent at M, w just in case:

  1. $M, w \models E^B \mu$
  2. $M, w \models \neg B \mu$
  3. $M, w \models \neg B \neg \alpha_i$ for all $1 \leq i \leq n$

- We can thus express that μ is on the agenda by the formula:

$$A\mu := E^B \mu \wedge \neg B \mu \wedge \neg B \neg \alpha_1 \wedge \cdots \wedge \neg B \neg \alpha_n$$

N.B.: It is important here that μ is assumed to be in minimal form

Latent issues

- Besides issues that are on the current agenda, there are also issues that are latent.
- These are issues that are currently resolved, but would enter the agenda if some current belief were to be given up.

Formal characterization

Suppose $M, w \models B\mu$, that is, μ is currently resolved for the agent.

Then we say that μ is a latent issue at M, w if there is some α s.t.:

- $M, w \models B\alpha$
- $M, w \models [\downarrow\alpha]\, A\mu$

Application 3: inquisitive contraction

- We saw in the astronomer example that contraction can result in the addition of a new question to the research agenda
- There are also cases in which no question is added to the agenda.

Example

(after O&W)

- You heard that your uncle Peter is in Australia for business.
- But then, while driving through town, you see someone looking very much like Peter walking on the other side of the street.
- You are not sure it's him, so you do not come to firmly believe that Peter is still in town.
- But you do contract your belief that he is in Australia.
- Nonetheless, since it is not part of your long term epistemic goals to know where Peter is, no question is added to your research agenda.

Application 3: inquisitive contraction

- Thus, we can make a distinction between inquisitive and non-inquisitive contraction.
- The former is more significant in light of the agent's long term epistemic goals.

Formal characterization

Suppose that $M, w \models B\alpha$. Then we say that contracting the belief α in M, w is inquisitive just in case there is an interrogative μ such that:

$$M, w \models \neg A\mu \wedge [\downarrow \alpha] A\mu$$

Conclusion

- We have presentend a logical framework to model:
  - The beliefs of a set of agents
  - Their epistemic goals
  - How these change in the process of inquiry
- The framework allows us to give fine-grained formal characterizations of a number of interesting notions:
  - pertinent beliefs
  - research agenda
  - inquisitive contraction
- Next on our own agenda is an investigation of the logic to which the framework gives rise.

Selected references

On the role of the research agenda in epistemic change
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Van Benthem (2007), *Journal of Applied Non-Classical Logics*.

Dynamic belief revision over multi-agent plausibility models
Baltag and Smets (2006), *LOFT*.

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Modalities in the realm of questions
Ciardelli (2014), *Advances in Modal Logic*.

See also: `www.illc.uva.nl/inquisitivesemantics`