Intuitionistic Logic

Exercise sheet 11

January 23, 2018

Exercise 1. [Heyting arithmetic]
Prove that $\vdash_{\mathsf{HA}} \forall x(x = 0 \vee \neg(x = 0))$

Exercise 2. [Disjunction and existence property]

- Prove that, if $\vdash_{\mathsf{HA}} \neg\varphi \to \exists x\psi(x)$, then $\vdash_{\mathsf{HA}} \neg\varphi \to \psi(\overline{n})$ for some n.

  Hint: use Smorynski's trick.

- Add a predicate $B(x)$ to the language of HA with the axiom $\exists x B(x)$. Does the resulting system have the disjunction property?