Conditionals: between language and reasoning

Class 6 - The problem of disjunctive antecedents

December 9, 2017

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- In general, the following entailment patterns seems valid. They are known as simplification of disjunctive antecedents.

$$\frac{A \vee B \mathrel{\Box\!\!\rightarrow} C}{A \mathrel{\Box\!\!\rightarrow} C}\ (\mathrm{SDA}_1) \qquad \frac{A \vee B \mathrel{\Box\!\!\rightarrow} C}{B \mathrel{\Box\!\!\rightarrow} C}\ (\mathrm{SDA}_2)$$

- In fact, it seems that we have the following equivalence:

$$A \vee B \mathrel{\Box\!\!\rightarrow} C \quad \equiv \quad (A \mathrel{\Box\!\!\rightarrow} C) \wedge (B \mathrel{\Box\!\!\rightarrow} C)$$

An argument for the validity of SDA: what has the speaker said?

(3) a. A: Last week, John went to London and to Paris. $L \wedge P$
b. B: That's not true: he didn't go to London.

B challenges A's claim $L \wedge P$ by denying L.
This makes good sense since $L \wedge P \models L$.

(4) a. A: Last week, John went to London or to Paris. $L \vee P$
b. B: #That's not true: he didn't go to London.

B challenges A's claim $L \vee P$ by denying L.
This seems to misunderstand what A said, since $L \vee P \not\models L$.

(5) A: If Thorpe or Wilson were to win, Britain would prosper.
B: That's not true: if Thorpe were to win, Britain would go bankrupt.

- Here, A claims $T \vee W \mathrel{\Box\!\!\to} P$
- B challenges A's claim by denying that $T \mathrel{\Box\!\!\to} P$.
- This seems to make perfect sense here.
- This suggests that what A says indeed implies that if Thorpe were to win, Britain would prosper.
- If SDA is valid, this is immediately explained:

$$T \vee W \mathrel{\Box\!\!\to} P \quad \models \quad T \mathrel{\Box\!\!\to} P$$

Compare this to a case like the following.

(6) A: If I had played in this round, I would have lost.
B: #That's not true: if you had played and you had had good cards, you would have won.

- Here, A claims $P \mathrel{\Box\!\!\rightarrow} L$.
- B challenges this claim by denying that $P \wedge G \mathrel{\Box\!\!\rightarrow} L$.
- This doesn't sound sensible: clearly, A did not claim $P \wedge G \mathrel{\Box\!\!\rightarrow} L$.
- This witnesses once more the failure of antecedent strengthening:

$$P \mathrel{\Box\!\!\rightarrow} L \quad \not\models \quad P \wedge G \mathrel{\Box\!\!\rightarrow} L$$

Problem: SDA is not valid in minimal change semantics

- If A and B are equally distant ($S_w^A = S_w^B$) then SDA goes through:

$$w \Vdash A \vee B \mathrel{\Box\!\!\rightarrow} C \quad\Longleftrightarrow\quad w \Vdash (A \mathrel{\Box\!\!\rightarrow} C) \wedge (B \mathrel{\Box\!\!\rightarrow} C)$$

- However, if B is more remote than A ($S_w^A \subset S_w^B$), then the closest $A \vee B$ worlds are just the closest A worlds

$$w \Vdash A \vee B \mathrel{\Box\!\!\rightarrow} C \quad\Longleftrightarrow\quad w \Vdash A \mathrel{\Box\!\!\rightarrow} C$$

- Is this right?
- Perhaps we only considered cases where the disjuncts are equidistant. Let's try to consider a case where one disjunct is more remote.

Consider the following scenario: the summer is over and you and I are visiting a farm. The owner of the farm is complaining about last summer's weather. To give us an example of its devastating effects, he points to the site where he used to grow huge pumpkins: there is a bunch of immature pumpkins and many ruined pumpkin plants. He then utters the counterfactual in (1):

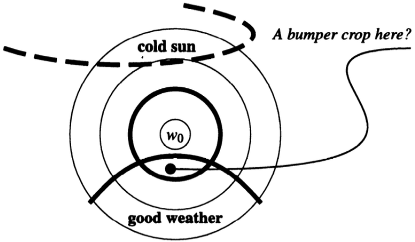
(1) If we had had good weather this summer or the sun had grown cold, we would have had a bumper crop.
(A variation on an example in Nute 1975.)

We conclude, right then, that there is something strange about this farmer. We have a strong intuition that the counterfactual in (1) is false: if we had had good weather this summer, he would have had a good crop, but we know for a fact—and we assume that the farmer does too—that if the sun had grown cold, the pumpkins, much as everything else, would have been ruined.

(Alonso-Ovalle 2009)

- Presumably, in order to contemplate the good-weather possibility G we need not take into account any major changes in the Solar System.

- So the closest $G \vee C$-worlds are just the closest G-worlds.



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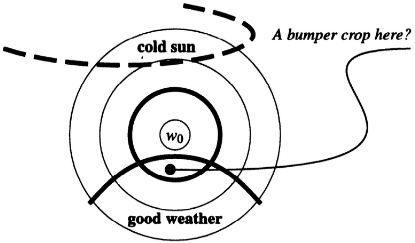

- Then the farmer's statement is predicted to be true:

(7) If we had had good weather or the sun had grown cold, we would have had a bumper crop.

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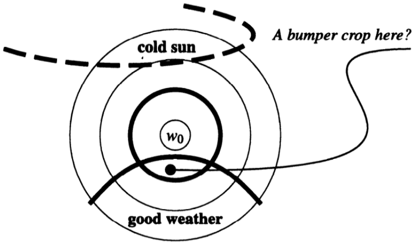

- Then the farmer's statement is predicted to be true:

(7) If we had had good weather or the sun had grown cold, we would have had a bumper crop.

- This is wrong. In assessing (7), we do not disregard cold-sun worlds. We take them into account, and that's why we don't judge (7) as true.

- Is this just a bug of minimal change semantics?
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- Is this just a bug of minimal change semantics?
  Can we patch up the theory in a simple way to get SDA?
- No!
- Minimal change semantics is formulated within intensional semantics.
- The extension of a sentence A is its truth-value in a particular world.
- Its intension is a function $\chi_A : W \to \{0, 1\}$ from worlds to extensions.
- This function can be identified with the set $|A|$ of worlds:

$$|A| = \{w \in W \mid \chi_A(w) = 1\} \qquad \chi_A(w) = \begin{cases} 1 & \text{if } w \in |A| \\ 0 & \text{otherwise} \end{cases}$$

- The truth-conditions of $A \mathrel{\Box\!\!\rightarrow} C$ depend on the intensions of A and C:

$$w \Vdash A \mathrel{\Box\!\!\rightarrow} C \iff \min(w, |A|) \subseteq |C|$$

- If $|A| = |B|$ then $|A \mathrel{\Box\!\!\rightarrow} C| = |B \mathrel{\Box\!\!\rightarrow} C|$.

- So, minimal change semantics validates the following principle, called substitution of equivalent antecedents.

$$\frac{A \mathrel{\Box\!\!\rightarrow} C \quad A \equiv B}{B \mathrel{\Box\!\!\rightarrow} C} \text{ (SEA)}$$

- Most instances of the principle are well-supported by intuition:

(8) If it hadn't rained for a fortnight, the crop would have been ruined.

(9) If it hadn't rained for two weeks, the crop would have been ruined.

(10) If I had not passed that exam, I wouldn't have graduated on time.

(11) If I had failed that exam, I wouldn't have graduated on time.

- But as Fine realized, (SDA) and (SEA) imply antecedent strengthening, which minimal change semantics was designed to invalidate:

$$\frac{A \mathrel{\Box\!\!\rightarrow} C}{A \wedge B \mathrel{\Box\!\!\rightarrow} C} \text{ (AS)}$$

- Then here is the proof of (AS):
  - $A \mathrel{\Box\!\!\rightarrow} C$ — assumption
  - $A \equiv (A \wedge B) \vee (A \wedge \neg B)$ — classical logic
  - $((A \wedge B) \vee (A \wedge \neg B)) \mathrel{\Box\!\!\rightarrow} C$ — by SEA
  - $A \wedge B \mathrel{\Box\!\!\rightarrow} C$ — by SDA

- The problem with SDA is more than a bug of minimal-change semantics: no intensional theory can validate SDA but not AS.

- So, if we want to validate SDA, we have only two options:
  1. Give up SEA
  2. Accept SA
- As invalidating SA was a core motivation, the natural option is 1.
- But we don't want to be too radical: (12) and (13) should be equivalent.

(12) If it hadn't rained for a fortnight,
the crop would have been ruined.

(13) If it hadn't rained for two weeks,
the crop would have been ruined.

- Alonso-Ovalle provides a way to vindicate SDA while retaining SEA in cases non involving disjunction.

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The suggestion goes back to Fine (1975):
"[...] each sentence refers to or indicates certain states-of-affairs. $\varphi \vee \psi$ would then refer to whatever states of affairs were referred to by φ or ψ [...] and the counterfactual $\varphi \mathrel{\Box\!\!\rightarrow} \psi$ would then be true if and only if each counterfactual $\chi \mathrel{\Box\!\!\rightarrow} \psi$ were true whenever χ singly referred to any state-of-affairs referred to by φ."

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- G refers to the state of affairs "good weather";
- C refer to the state of affairs "cold sun";
- $G \vee C$ refers to two states of affairs, "good weather" and "cold sun";
- $G \vee C \mathrel{\Box\!\!\rightarrow} B$ true $\iff$ $G \mathrel{\Box\!\!\rightarrow} B$ true and $C \mathrel{\Box\!\!\rightarrow} B$ true.

Alternative semantics (originating with Hamblin 73)

- All expressions denote sets of objects — called alternatives.
- Sentential clauses denote sets of propositions.
- Basic clauses denote the singleton set of their standard proposition.

$$G = \text{We had good weather} \qquad [\![G]\!] = \{\,|G|\,\}$$

- Ordinary operators are lifted to operate point-wise on each element.
  - Example: in standard intensional semantics, $\wedge$ performs intersection:

    $$|A \wedge B| = |A| \cap |B|$$

  - In alternative semantics, this is lifted to point-wise intersection:

    $$[\![A \wedge B]\!] \;=\; \{p \cap q \mid p \in [\![A]\!] \text{ and } q \in [\![B]\!]\}$$

  - When A and B are basic clauses, this gives the expected results:

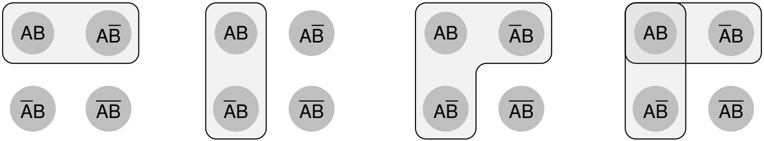
    $$\begin{aligned} [\![A \wedge B]\!] &= \{\,p \cap q \mid p \in \{|A|\} \text{ and } q \in \{|B|\}\,\} \\ &= \{\,|A| \cap |B|\,\} \\ &= \{\,|A \wedge B|\,\} \end{aligned}$$

Disjunction à la Alonso-Ovalle

$[\![A \vee B]\!] = [\![A]\!] \cup [\![B]\!]$

If A and B are basic clauses:

- $[\![A]\!] = \{|A|\}$
- $[\![B]\!] = \{|B|\}$
- $[\![A \vee B]\!] = \{|A|\} \cup \{|B|\} = \{|A|, |B|\}$



(a) A (b) B (c) $A \vee B$ classical (d) $A \vee B$ in AltSem

Conditionals à la Alonso-Ovalle

- An antecedent doesn't always provide a single assumption.
- When it has multiple alternatives, each is processed as a separate assumption.
- The counterfactual is true if the consequent follows on each of these.

$$w \Vdash A \,\Box\!\!\rightarrow C \iff \forall p \in [\![A]\!] : \min_w(p) \subseteq |C|$$

- The counterfactual then denotes a singleton alternative—the set of worlds where it is true:

$$[\![A \,\Box\!\!\rightarrow C]\!] = \{|A \,\Box\!\!\rightarrow C|\}$$

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- NB: this actually differs from A.-O.'s proposal when C has multiple alternatives; A.-O.'s proposal seems to make wrong predictions there, but I will set this aside.

Vindicating SDA

Suppose A, B, C are simple clauses.

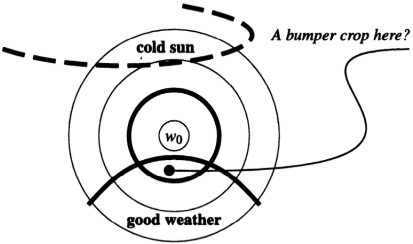
$$
\begin{aligned}
w \Vdash A \vee B \mathrel{\Box\!\!\rightarrow} C \quad &\Longleftrightarrow \quad \forall p \in [\![A \vee B]\!] : \min_w(p) \subseteq |C| \\
&\Longleftrightarrow \quad \forall p \in \{|A|, |B|\} : \min_w(p) \subseteq |C| \\
&\Longleftrightarrow \quad \min_w(|A|) \subseteq |C| \text{ and } \min_w(|B|) \subseteq |C| \\
&\Longleftrightarrow \quad w \Vdash A \mathrel{\Box\!\!\rightarrow} C \text{ and } w \Vdash B \mathrel{\Box\!\!\rightarrow} C \\
&\Longleftrightarrow \quad w \Vdash (A \mathrel{\Box\!\!\rightarrow} C) \wedge (B \mathrel{\Box\!\!\rightarrow} C)
\end{aligned}
$$

So we get the following equivalence, which implies SDA:

$$A \vee B \mathrel{\Box\!\!\rightarrow} C \;\equiv\; (A \mathrel{\Box\!\!\rightarrow} C) \wedge (B \mathrel{\Box\!\!\rightarrow} C)$$

Next is rightly predicted to be false.

(14) If we had had good weather or the sun had grown cold, we would have had a bumper crop.



- In general, this account breaks SEA (more next time).
- For instance, the following have the same truth-conditions:

(15) a. This match is struck. S
b. This match is struck and wet, or it is struck and dry. $(S \wedge W) \vee (S \wedge \neg W)$

- However, in AltSem they get different alternatives:

(16) a. $[\![S]\!] = \{|S|\}$
b. $[\![(S \wedge W) \vee (S \wedge \neg W)]\!] = \{|S \wedge W|, |S \wedge \neg W|\}$

- This leads to different truth-conditions when these clauses are embedded in a counterfactual.

(17) a. If this match were struck, it would light.

b. $w \Vdash S \mathrel{\Box\!\!\rightarrow} L \iff \min_w(|S|) \subseteq |L|$

(18) a. If this match were struck and were wet,
or it were struck and were dry, it would light.

b. $w \Vdash S \mathrel{\Box\!\!\rightarrow} L \iff \min_w(|S \wedge W|) \subseteq |L|$ and $\min_w(|S \wedge \neg W|) \subseteq |L|$

- In a normal scenario, (17-a) will be true and (18-a) false – against SEA.
- This prediction seems in accordance with intuitions.

Conservativity over minimal change semantics

- When A is a disjunction-free antecedent, $[\![A]\!] = \{\, |A| \,\}$.

$$
\begin{aligned}
w \Vdash A \mathrel{\Box\!\!\rightarrow} C \quad &\iff \quad \forall p \in \{\, |A| \,\} : \min_w(p) \subseteq |C| \\
&\iff \quad \min_w(|A|) \subseteq |C|
\end{aligned}
$$

- This shows that A.-O.'s account is a rather conservative revision: when disjunction is not around, it boils down to Lewis's semantics.

- Thus, the semantics still invalidates AS.
  (19-a) does not entail (19-b):

(19) a. If I had played, I would have lost.
     b. If I had had good cards and I had played, I would have lost.

- Thus, the semantics still invalidates AS.
  (19-a) does not entail (19-b):

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- Moreover, for $\vee$-free antecedents, it respects SEA.
  Thus, it accounts for intuitive equivalences such as (20) $\equiv$ (21):

(20) If it hadn't rained for a fortnight, the crop would have been ruined.

(21) If it hadn't rained for two weeks, the crop would have been ruined.

Counterexamples to SDA? (McKay & van Inwagen 77)

(22) If Spain had sided with the Axis or the Allies in WWII, it would have sided with the Axis.

(23) If Spain had sided with the Allies, she would have sided with the Axis.

- Clearly, (22) does not imply (23), contrary to SDA and to Alonso-Ovalle's account.
- Plain minimal change semantics seems to make the right predictions: in the closest worlds where Spain takes sides, she sides with the Axis; thus (22) is true, but obviously (23) is false.

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- Clearly, (22) does not imply (23), contrary to SDA and to Alonso-Ovalle's account.
- Plain minimal change semantics seems to make the right predictions: in the closest worlds where Spain takes sides, she sides with the Axis; thus (22) is true, but obviously (23) is false.
- However, there is a problem with this.

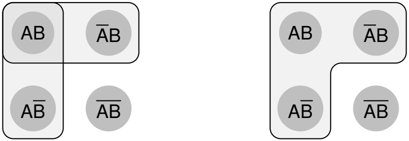
(24) If Spain had sided with the Axis or the Allies in WWII, Hitler would have been pleased.

- Intuitively, (24) is not true.
- But if the closest antecedent worlds are ones where Spain joins the axis, minimal change semantics would predict (24) to be true.
- But if the closest antecedent worlds are ones where Spain joins the axis, minimal change semantics would predict (24) to be true.
- It seems that failures of SDA occur only with very special consequents. . . why?

Alonso-Ovalle's proposal

- It is possible to merge the alternatives for the antecedent into one via an operator $\exists$ that he calls existential closure.

$$[\![\exists A]\!] = \{\bigcup [\![A]\!]\}$$



(e) $A \vee B$ (f) $\exists(A \vee B)$

- If $\exists$ is inserted, we retrieve the predictions of minimal change semantics.

- However, inserting $\exists$ is possible only as a last resource, to avoid interpreting (24) as a (contextual) contradiction.

(25) If Spain had sided with the Axis or the Allies in WWII, it would have sided with the Axis.

- If Spain siding with the Allies is an entertainable supposition, and if it is impossible to side with both the Axis and the Allies, the SDA reading of (25) is contradictory.
- This is what motivates the insertion of $\exists$.
- On the other hand, the SDA reading of (26) is consistent, so $\exists$ is not inserted and we get a standard reading.

(26) If Spain had sided with the Axis or the Allies in WWII, Hitler would have been pleased.

- But not all such examples are contradictory under the SDA reading.
- For instance, intuitively (27-a) does not entail (27-b).

(27) a. If I were to invite Alice or Bob, I would invite Alice.
b. If I were to invite Bob, I would invite Alice.

- In this case the SDA reading of (27-a) is equivalent to (28).

(28) If I were to invite Bob, I would invite Alice.

- This doesn't seem contradictory: it is possible to invite both.

- Moreover, there are other puzzling observations about such sentences.
- While (29) sounds ok, the variants in (30) sound very odd.

(29) If Spain had sided with the Axis-or-the-Allies,
it would have sided with the Axis.

(30) a. If Spain had sided with the Axis OR with the Allies,
it would have sided with the Axis.

b. If Spain had sided with the Axis or if it had sided with the Allies,
it would have sided with the Axis.

c. If Spain had sided with the Axis, or had betrayed the Axis and
sided with the Allies, it would have sided with the Axis.

- This different is unexpected on both the standard view and A.-O.'s view.