Conditionals: between language and reasoning

Class 7 - Breaking de Morgan's law in counterfactual antecedents +
Lifting conditionals to inquisitive semantics

December 8, 2017

Part I

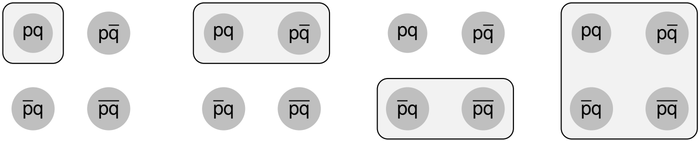
Inquisitive semantics: a very short introduction

- Inquisitive semantics (Ciardelli,Groenendijk&Roelofsen 2013) is an approach to semantics designed to deal uniformly with statements and questions.
- Standardly, the fundamental semantic notion is that of truth at a world.
- The meaning of a sentence φ can be identified with the set $|\varphi|$ of worlds where it is true.
- This works for statements like (1-a,b), but not for questions like (2-a,b):

(1) a. Alice likes Bob.
    b. Paris is the capital of France.

(2) a. Does Alice like Bob?
    b. What is the capital of France?

- InqSem starts from a more information-oriented perspective: the basic semantic notion is support wrt an information state.
- An info state s is modeled extensionally as a set of possible worlds: the worlds compatible with the relevant information.



(a) s_1 (b) s_2 (c) s_3 (d) s_4

- Statement α is supported if the given information implies that α is true.

$$s_i \models p \iff i = 1, 2$$

- Question μ is supported if the given information resolves μ.

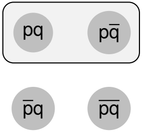
$$s_i \models ?p \iff i = 1, 2, 3$$

- Support is persistent:

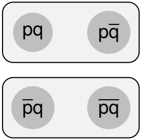
$$s \models \varphi \text{ and } t \subseteq s \quad \Rightarrow \quad t \models \varphi$$

- The alternatives for a formula are the maximal states that support it:

$$\text{Alt}(\varphi) = \{s \mid s \models \varphi \text{ and there is no } t \supset s \text{ with } t \models \varphi\}$$



(e) p



(f) $?p$

General entailment

$\varphi_1, \ldots, \varphi_n \models \psi \iff \forall s : s \models \varphi_i$ for $1 \leq i \leq n$ implies $s \models \psi$.

Examples

- Resolution:

$$p \to q,\ p \ \models \ ?q$$

- Dependency:

$$p \leftrightarrow q,\ ?p \ \models ?q$$

General entailment

$\varphi_1, \ldots, \varphi_n \models \psi \iff \forall s : s \models \varphi_i$ for $1 \leq i \leq n$ implies $s \models \psi$.

Examples

- Resolution:

$$p \rightarrow q,\ p \ \models \ ?q$$

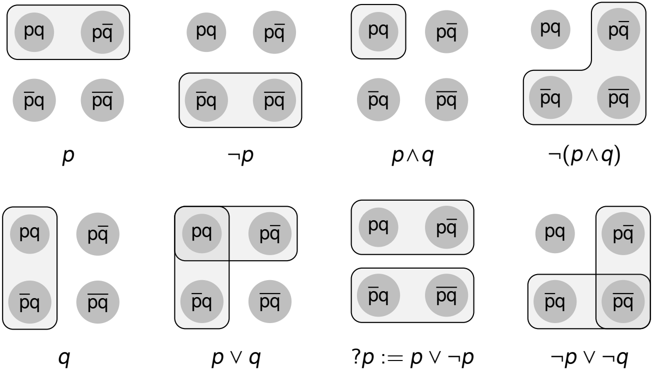
- Dependency:

$$p \leftrightarrow q,\ ?p \ \models ?q$$

More in the course next semester!

Inquisitive semantics comes with a theory of propositional connectives, motivated by logical/algebraic considerations:

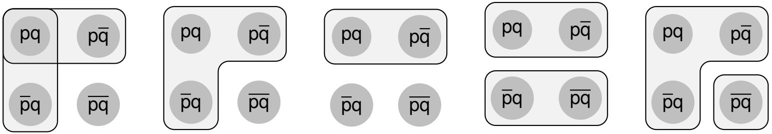
- $s \models \varphi \wedge \psi \iff s \models \varphi$ and $s \models \psi$
- $s \models \varphi \vee \psi \iff s \models \varphi$ or $s \models \psi$
- $s \models \neg\varphi \iff$ for no consistent $t \subseteq s : t \models \varphi$



This theory validates intuitionistic logic (and some more principles).
NB de Morgan's law is invalid: $\neg(p \wedge q) \not\equiv \neg p \vee \neg q$.

We also have two projection operators:

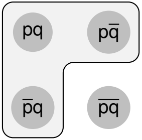
- $!\varphi := \neg\neg\varphi$ collapses alternatives into one
- $?\varphi := \varphi \vee \neg\varphi$ adds $\overline{\bigcup \mathrm{Alt}(\varphi)}$ as an alternative



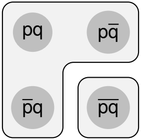
$p \vee q$ $!(p \vee q)$ p $?p$ $?!(p \vee q)$

One gain: a uniform account of connectives in statements and questions.

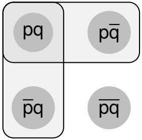
(3)
a. Mark went to London or to Paris. $!(p \vee q)$
b. Did Mark go to either London or Paris? $?!(p \vee q)$
c. Did Mark go to London, or to Paris? $p \vee q$



$!(p \vee q)$

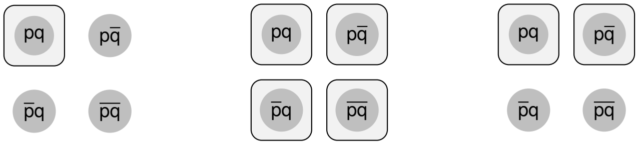


$?!(p \vee q)$



$p \vee q$

(4) a. Alice likes Bob and he likes her. $p \wedge q$
b. Does Alice like Bob, and does he like her? $?p \wedge ?q$
c. Alice likes Bob, but does he like her? $p \wedge ?q$



$p \wedge q$ $?p \wedge ?q$ $p \wedge ?q$

This extends to other operators. With a single clause for K we can analyze:

(5) a. Alice knows that Bob likes her. $K_a p$
b. Alice knows whether Bob likes her. $K_a ?p$

- So, inquisitive semantics provides a notion of meaning which is more fine-grained than the standard truth-conditional one.
- The finer grain is certainly needed to deal with questions.
- Is it also useful to analyze statements, or could we stick with truth-conditions as far as they are concerned?
- Claim (following up on the last class): counterfactuals call for a fine-grained semantic representation of antecedents.
- This representation should be one that breaks de Morgan's law:

$$\neg p \vee \neg q \not\equiv \neg(p \wedge q)$$