Questions and Dependency in Logic

Take-home exam

July 17, 2018

Solutions should be submitted in pdf format, together with your term paper, to: ivano.ciardelli@lmu.de

The deadline is September 17th 2018.

Make sure to be explicit and precise, and please structure your answers in a way that makes them easy to follow.

Exercise 1. [Foundations]
Explain in your own words the connection between the relation of dependency and the implication connective in inquisitive logic.

Exercise 2. [Propositional dependencies and proofs involving questions]
Consider the following "whodunnit" setting: Alice was found dead in the library. The police think it was a suicide, but the detective suspects it might be murder. She is convinced that, if it was in fact murder, then the murderer must be either Bob or Charlie, since they are the only ones who had motive and opportunity. However, Bob has a strong alibi for the hours before noon, while Charlie has one for the hours after noon. From the detective's point of view, then, the question of whether Alice's death took place before or after noon determines the question of who is the culprit if it was murder.

- *Task 1.* Formalize this dependency as an instance of entailment in InqB, involving some statement assumptions (some of the facts given in the story) a question assumption (whether the death took place before or after noon) and a question conclusion (whether B or C is the culprit, if it was murder).
- *Task 2.* Prove this entailment using the natural deduction system for InqB. Omit the inference steps that just involve classical formulas, as these are just inference steps in classical logic.

Exercise 3. [Propositional inquisitive logic]
Let InqB_p be the p-fragment of InqB, i.e., the fragment consisting of formulas containing only one propositional letter p. How many equivalence classes of formulas are there in this fragment? Justify your answer and draw the set of these equivalence classes ordered by entailment.

Exercise 4. [Questions and partitions]
Suppose that, instead of introducing questions in CPC by means of $\mathbin{\rotatebox[origin=c]{-90}{$\geq$}}$, we do so by adding ? as a primitive connective, and consider the following restricted set $\mathcal{L}_{\mathsf{A}}$ of formulae, where $\alpha \in \mathcal{L}_c$:

$$\varphi \ ::= \ \alpha \mid ?\alpha \mid \varphi \wedge \varphi$$

(notice that only conjunction, and not implication, is allowed over questions). Suppose ? is interpreted by the natural support condition:

$$s \models ?\alpha \iff s \models \alpha \text{ or } s \models \neg\alpha$$

Call this system InqA (notice that InqA is a sub-system of InqB).

Let us say that a formula φ in InqB is *partitional* if for all models M there is an equivalence relation $\sim_\varphi^M$ on a subset of W such that for all $s \subseteq W$:

$$s \models \varphi \iff \forall w, w' \in s : w \sim_\varphi^M w'$$

(Equivalently, a formula is partitional iff its set of alternatives forms a partition of a subset of the universe; but you are not required to prove this.)

1. Show that all $\varphi \in \mathcal{L}_{\mathsf{A}}$ are partitional.

2. Show that all partitional formulas φ in InqB are equivalent to a $\varphi' \in \mathcal{L}_{\mathsf{A}}$. So, InqA is exactly the partitional fragment of InqB.

   Hint. Notice that we have a canonical model $\omega_{\mathcal{P}} = \langle W_\omega, V_\omega \rangle$, where the worlds $w \in W_\omega$ are propositional valuations $w : \mathcal{P} \to \{0, 1\}$, and V_ω is defined in the obvious way. Every inquisitive non-entailment is witnessed by this model (you may assume this in your solution). Given a partitional formula φ, use the relation $\sim_\varphi^{\omega_{\mathcal{P}}}$ to construct a formula $\psi \in \mathcal{L}_{\mathsf{A}}$ which agrees with φ on every state in M_ω.