Questions and Dependency in Logic: suggestions for the final essay

Ivano Ciardelli

June 28, 2018

The idea for the final essay is to encourage you to engage with an open problem in inquisitive logic. The subject is particularly suitable in this respect, since it is currently under development, and many questions are still unexplored.

In order to get a good grade, it is not required that you make substantial progress on the problem—though of course, that would be an ideal outcome. What is crucial that you present your research question in a clear form, and describe your work on the problem in a way that demonstrates command of the basic notions and results that we have seen in the course. An original contribution is required in order to achieve the highest grades (1–1.7). The essay should be approximately 10-15 pages long.

Below is a list of possible research questions, with pointers to the relevant literature. Each of them concern an outstanding issue, and thus provides an opportunity to make a novel contribution. You are also welcome to come up with your own research question: in that case, please get in touch with me in advance, so I can give suggestions and point you to relevant literature.

Overview:

1. Inquisitive logic over variable domains
2. Interpretations of inquisitive modal logic
3. $\Diamond$ in inquisitive modal logic
4. Alternative proof systems for InqB
5. Generalizing truth-conditionality
6. Public modalities in inquisitive epistemic logic
7. Adding questions to non-classical logic
8. Modal logic and scalable modal formulas
9. Questions and supervenience
10. IDEL and Moorean phenomena
11. Inquisitive dynamics beyond public utterance

Topic 1. [Inquisitive logic over variable domains]
For simplicity, in inquisitive first-order logic we assume to have a fixed domain of individuals shared among all possible worlds. This amounts to assuming that there is no uncertainty about the domain of quantification (except insofar as it results from uncertainty about identity). You could consider how to set up a version of first-order inquisitive logic where different worlds have possibly different domains. Among the issues that can be addressed are: what kind of questions/logical phenomena can be modeled in this setting that cannot be modeled in the constant domain setting; what repercussions this generalization has on the logic of the system; and whether this extension can be simulated in constant domain models if the language is extended with an existence predicate.

Relevant literature:

- Väänänen (2014): an implementation of variable domains in the context of dependence logic.
- Troelstra and van Dalen (1988), §2.5: Kripke semantics for intuitionistic logic over models with variable domains.

Topic 2. [Interpretations of inquisitive modal logic]
We have looked at inquisitive modal logic with a particular interpretation in mind, namely, the inquisitive-epistemic one. However, just like Kripke-style modal logic, inquisitive modal logic in principle allows for different intended interpretations, depending on what we take the inquisitive state $\Sigma(w)$ to encode. You could think of a different interpretation for inquisitive modal logic and explore the significance of various operators in that setting as well as the natural conditions on the map(s) Σ (and corresponding modal axioms) which are suggested by that concrete interpretation. Since inquisitive epistemic models are a special class of neighborhood models, some inspiration might come from looking at interpretations of neighborhood models.

Relevant material:

- Pacuit (2017): an introduction to neighborhood semantics for modal logic.

Topic 3. [$\Diamond$ in inquisitive modal logic]
Inquisitive modal logic comes with two modalities, $\Box$ and $\boxplus$, which extend the modality $\Box$ of standard modal logic to questions. What about the existential modality, '$\Diamond$'? Of course, we can define $\Diamond\varphi := \neg\Box\neg\varphi$ (or, equivalently, as $\neg\boxplus\neg\varphi$). While this operator has the standard meaning when it is applied to a statement, it is not very interesting when applied to questions (why?). You could consider alternative ways of extending $\Diamond$ to questions. Among the interesting issues to be investigated are: what kind of modal properties can be expressed by applying the resulting modality to questions? For what interpretations of modal logic are these properties interesting? What is the logic of these operators?

Relevant material:

- you may look at Aloni (2007) and at §6.8 of Ciardelli (2016) for inspiration.

Topic 4. [Alternative proof systems for InqB]
In the course, we have axiomatized propositional inquisitive logic InqB and some of its extensions by means of natural deduction systems. You could try to design other kinds of proof systems for InqB—in particular, tableaux systems or sequent calculi . You could show soundness and completeness for your calculus, and possibly show how it can be used as a decision procedure and—in case of invalidity—to construct a counter-model.

Relevant material:

- Chagrov and Zakharyaschev (1997), Section 2.4: an introduction to tableaux systems for intuitionistic logic.
- Frittella *et al.* (2016): a multi-type display calculus for inquisitive logic.

Topic 5. [Generalizing truth-conditionality]
In the course we have looked in detail at the property of *truth-conditionality*. We saw that the set of truth-conditional formulas contains atoms and it is closed under certain operations (e.g., if α is truth-conditional, so is $\varphi \to \alpha$ for all φ). We also saw that truth-conditional formulas have special logical features, such as double negation elimination and the local split property.

Truth-conditionality is a special case for $n = 1$ of a more general property called n-coherence. φ is n-coherent if for every model M and state s:

$$M, s \models \varphi \iff M, t \models \varphi \text{ for all } t \subseteq s \text{ of cardinality at most } n$$

You could study the properties of the fragment $\mathcal{L}_n$ consisting of n-coherent formulas (either in the propositional setting, or in the first-order setting, or both). Under what operations it is closed? What logical/model-theoretic features do n-coherent formulas have?

Relevant material:

- Ciardelli (2009), Ch. 4: a proof that for any natural number n there are $n + 1$-coherent formulas that are not n coherent.
- Kontinen (2010): a study of n-coherence in the dependence logic setting.

Topic 6. [Public modalities in inquisitive epistemic logic]
Common knowledge is a key notion in epistemic logic. In *inquisitive* epistemic logic, the common knowledge construction is extended to the construction of a public inquisitive-epistemic state that encompasses both common knowledge and public issues. This supports the interpretation of group modalities $\Box_*$ and $\boxplus_*$ which generalize the common knowledge modality of epistemic logic. You could work on a sound and complete axiomatization of the logic of these modalities, combining the standard proof of completeness for epistemic logic with common knowledge with the completeness proof for inquisitive modal logic.

Relevant material:

- Ciardelli (2016), §7.2.4.
- van Ditmarsch *et al.* (2007), Ch. 2 (discussion of common knowledge) and Ch. 7 (axiomatization).

Topic 7. [Adding questions to non-classical logics]
In the course we have looked at how systems of classical logic can be enriched with questions. But questions can also be added on the basis of other logics of statements. For instance, Ciardelli *et al.* (2017) study the addition of questions to intuitionistic propositional logic, while Punčochář (2018) provides a general framework that allows us to add questions to many kinds of non-classical propositional logics. A possible task along this line is to generalize these approaches beyond the propositional realm, to first-order logic or modal logic.

Relevant material:

- Ciardelli, Iemhoff, and Yang (2017), on adding questions and dependencies to intuitionistic logic.
- Punčochář (2018), on a general method for adding questions to a variety of non-classical logics (this paper is not available online, but I have permission of the author to share it on request).

Topic 8. [Modal logic and scalable modal formulas]
With every normal modal logic L we can associate an inquisitive extension InqBL, defined as the inquisitive Kripke modal logic of the class of frames defined by L (see Section 6.4 of Ciardelli (2016)). Now let α be a formula of standard modal logic. If α is valid in a logic L, then α is also valid in InqBL (by conservativity of the semantics), and so are all classical substitution instances of α (where atoms are replaced by classical formulas). However, it might be the case that replacing some atoms with questions results in formulas α^* which are invalid in InqBL.

Let us say that α is *scalable* if this cannot happen, i.e., if whenever α is valid in L, it is also schematically valid in InqBL. Here are two examples:

- Ex. 1: the modal logic $\mathsf{K4}$ is axiomatized by the formula $\Box p \to \Box\Box p$, which defines the class of transitive frames. In the inquisitive extension of this logic, $\mathsf{InqBK4}$, all formulas of the form $\Box\varphi \to \Box\Box\varphi$, for instance $\Box ?p \to \Box\Box ?p$, are logically valid. So, the formula $\Box p \to \Box\Box p$ is scalable.
- Ex. 2: the modal logic KT is axiomatized by the formula $\Box p \to p$, which defines the class of reflexive frames. However, in the inquisitive extension of this logic, InqBKT, the formula $\Box ?p \to ?p$ is not logically valid. Thus, the formula $\Box p \to p$ is not scalable.

Question: which modal formulas are scalable? An ideal answer to this question would provide a characterization of scalable formulas. However, it would already be interesting to determine for some of the most important modal axioms whether or not they are scalable.

Relevant material:

- Ciardelli (2016), §6.4, for relevant definitions and some initial observations.
- Humberstone (2018), §7, for a discussion of issues very close to scalability.

Topic 9. [Questions and supervienience]
In the philosophical literature, a property Q is said to supervene on properties $P_1, \ldots, P_n$ if two individuals cannot differ with respect to Q while being identical with respect to $P_1, \ldots, P_n$. As Humberstone (1993, 2002, 2018) has shown, the relation of supervienience can be analyzed as a kind of entailment relation which preserves agreement rather than truth, and it is tightly related to the notion of functional dependency. Since, as we discussed in class, functional dependency is a special case of question entailment in context, it is clear that there is a link between questions and supervenience. You might try to spell out more explicitly what this link is, and see whether the ideas of inquisitive logic suggest a different perspective on, and perhaps a generalization of, existing work on supervienience. In particular, you might go beyond the case of propositions and look at supervenience of properties (in an intensional setting).

Relevant material:

- Humberstone (1993, 2002, 2018).

Topic 10. [IDEL and Moorean phenomena]
A formula φ in public announcement logic (PAL) is *successful* if in all models, a public announcement of φ leads to a situation where φ is common knowledge. Not all formulas are successful: in fact, the Moore formula $\varphi_M = p \land \neg K_a p$ is not just unsuccessful, but also *self-falsifying*: an announcement of φ_M produces a situation where φ_M is false. An interesting task is that of studying the extent of Moorean phenomena in IDEL (e.g., self-resolving questions, or questions undermining their own presupposition), defining a notion of success in IDEL and studying which formulas have this property.

Relevant material:

- see Holliday and Icard (2010), and the references therein, for a detailed study of successful and unsuccessful formulas in PAL.
- see Ciardelli (2016), §8.4 for a fragment of IDEL all of whose formulas are in a natural sense "successful" in the inquisitive setting.

Topic 11. [Inquisitive dynamics beyond public utterance]
In order to model many communication scenarios, we need to go beyond public announcements, to a framework where we can also model private actions. Two influential logics designed for this purpose are the *Epistemic Action Logic* (EAL) of van Ditmarsch (2000, 2002, 2003), and the *Action Model Logic* of Baltag, Moss, and Solecki (1998). An inquisitive generalization of the latter approach was developed and studied by van Gessel (2016). You could try to develop an inquisitive generalization of *Epistemic Action Logic*, and demonstrate the workings of the resulting dynamic logic with some concrete examples.

Relevant material:

- van Ditmarsch *et al.* (2007): an introduction to EAL.
- van Ditmarsch (2000, 2002, 2003): the original literature on EAL.
- van Gessel (2016): an inquisitive extension of *Action Model Logic*.

References

Aloni, M. (2007). Free choice, modals and imperatives. *Natural Language Semantics*, **15**(1), 65–94.

Baltag, A., Moss, L. S., and Solecki, S. (1998). The logic of public announcements, common knowledge, and private suspicions. In *Proceedings of the 7th Conference on Theoretical Aspects of Rationality and Knowledge*, pages 43–56. Morgan Kaufmann Publishers.

Chagrov, A. and Zakharyaschev, M. (1997). *Modal Logic*. Oxford University Press.

Ciardelli, I. (2009). Inquisitive semantics and intermediate logics. MSc Thesis, University of Amsterdam.

Ciardelli, I. (2016). *Questions in logic*. Ph.D. thesis, Institute for Logic, Language and Computation, University of Amsterdam.

Ciardelli, I., Iemhoff, R., and Yang, F. (2017). Questions and dependency in intuitionistic logic. Manuscript, under review.

van Ditmarsch, H. (2000). *Knowledge Games*. Ph.D. thesis, University of Groningen.

van Ditmarsch, H. (2002). Descriptions of game actions. *Journal of Logic, Language and Information*, **11**(3), 349–365.

van Ditmarsch, H. (2003). The russian cards problem. *Studia Logica*, **75**(1), 31–62.

van Ditmarsch, H., van der Hoek, W., and Kooi, B. (2007). *Dynamic Epistemic Logic*. Springer.

Frittella, S., Greco, G., Palmigiano, A., and Yang, F. (2016). A multi-type calculus for inquisitive logic. In *International Workshop on Logic, Language, Information, and Computation*, pages 215–233. Springer.

Holliday, W. H. and Icard, T. F. I. (2010). Moorean phenomena in epistemic logic. In L. Beklemishev, V. Goranko, and V. Shehtman, editors, *Advances in Modal Logic (AiML)*, pages 178–199, London. College Publications.

Humberstone, L. (1993). Functional dependencies, supervenience, and consequence relations. *Journal of Logic, Language and Information*, **2**(4), 309–336.

Humberstone, L. (2002). The modal logic of agreement and noncontingency. *Notre Dame Journal of Formal Logic*, **43**(2), 95–127.

Humberstone, L. (2018). Supervenience, dependence, disjunction. *Logic and Logical Philosophy*.

Kontinen, J. (2010). *Coherence and complexity in fragments of dependence logic*. Ph.D. thesis, Institute for Logic, Language and Computation, University of Amsterdam.

Pacuit, E. (2017). *Neighborhood Semantics for Modal Logic*. Springer. forthcoming.

Punčochář, V. (2018). Substructural inquisitive logics. Manuscript, under review.

Troelstra, A. and van Dalen, D. (1988). *Constructivism in mathematics, an introduction, Volume 1*, volume 121 of *Studies in Logic and the Foundations of Mathematics*. North-Holland.

Väänänen, J. (2014). Multiverse set theory and absolutely undecidable propositions. *Interpreting Gödel: Critical Essays*, page 180.

van Gessel, T. (2016). Action models in inquisitive logic. MSc thesis, University of Amsterdam.