Exercises. Prove that the following functions are recursive:

1. exponential:
$$\exp(x, y) = x^y$$

2. n-ary sum, for any $n \geq 2$:
$$\mathrm{sum}_n(x_1, \ldots, x_n) = x_1 + \cdots + x_n$$

3. predecessor:
$$\mathrm{pr}(x) = \begin{cases} x - 1 & \text{if } x \geq 1 \\ 0 & \text{if } x = 0 \end{cases}$$

4. asymmetric difference:
$$\mathrm{ad}(x, y) = \begin{cases} x - y & \text{if } x \geq y \\ 0 & \text{if } x < y \end{cases}$$

5. symmetric difference:
$$\mathrm{sd}(x, y) = \begin{cases} x - y & \text{if } x \geq y \\ y - x & \text{if } x < y \end{cases}$$

6. half:
$$\mathrm{half}(x) = \begin{cases} x/2 & \text{if } x \text{ is even} \\ \text{undefined} & \text{if } x \text{ is odd} \end{cases}$$