**Dynamic Epistemic Logic**

# Inquisitive Action Models

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**Abstract**

Inquisitive Dynamic Epistemic Logic (IDEL) is a public announcement logic in which not only statements, but also questions can be publicly uttered. So far, public utterance is the only dynamic operation defined in IDEL. To also make utterances possible of which not all agents have full knowledge, we define action models and update procedures for Inquisitive Epistemic Logic. We show that action models can be lifted to the inquisitive setting to also model questions and issues, instead of just information and knowledge.

# 1 Introduction

## 1.1 Updating with a question

Consider Anne and Bill. Last week, Anne was not feeling well, so she visited the doctor, who performed a test. The doctor has informed her that in a week she can expect a letter containing one of three messages: either the test result was positive ($p$), the result was negative ($\neg p$) or the doctor does not have enough information yet and requests more information from Anne ($?q$). Both Anne and Bill are aware of these options. When the letter arrives, Bill sees that Anne opens it and reads the message. At that moment, Anne knows the content of the message, but Bill does not.

In dynamic epistemic logic, situations like these can be represented in a model: when Anne reads the message, the model is updated in a way that represents that she can now distinguish between worlds where $p$ and worlds where $\neg p$. In contrast, Bill can still not distinguish $p$ and $\neg p$ worlds.

In this example, however, the message might be a question: if Anne reads the question $?q$, it will become one of her goals to answer it, which can be done in two ways: finding out that $q$ is true or that $q$ is false. Bill currently does not have this goal, because he does not know that the letter said $?q$. But as he is interested in Anne's well being, he has another goal: to find out what the letter said.

Clearly, the message was not a public announcement, because Bill has not read it. It is also not a message that can be encoded in standard epistemic logic, as the message might have inquisitive content. To capture such a situation in an epistemic model, the familiar ways of updating epistemic models have to be refined.

## 1.2 Two extensions of PAL

In Ciardelli & Roelofsen (2015), an Inquisitive Dynamic Epistemic Logic (IDEL) is developed, which is refined in Ciardelli (2016)[1]. It is Inquisitive Epistemic Logic (IEL) made dynamic by adding a public utterance operator. A public utterance is the inquisitive counterpart of public announcement: when a statement is announced, all worlds in which the statement is false are dropped, like in Public Announcement Logic (PAL). However, in IDEL a public utterance can also be a question. In that case, all agents that do not have enough information to resolve it, will entertain the question in the updated model: they will make it their goal to resolve it.

Because IDEL can encode only *public* utterances, all agents are always aware of the content of the utterance. As a result they will all learn the new information or entertain the new questions it contains. In standard, non-inquisitive DEL, it has long been possible to encode more actions than just public announcements. In Action Model Logic (AML) (Baltag et al., 1998), all possible actions are represented in an action model, which also encodes the knowledge each agent has about which action is happening. In this system, it can be the case that some agent learns some new information, while others are not sure what information this is. Such a situation results in a new model in which it is not necessarily the case that all agents drop the same worlds.

[1] In this paper, we follow the system as developed in the latter.

This means we now have two extensions of PAL: on the one hand, IDEL makes PAL more general by encoding the issues of agents, thereby making public questions possible. On the other hand, AML makes PAL more general by allowing not only public announcements, but also other epistemic actions. However, to model situations like our example we need a mix of both: a model that can encode issues, questions and multiple epistemic actions. The goal of this paper is to come up with a system that can encode the epistemic actions familiar from AML, but in which actions can have inquisitive content as well as informative content.

The rest of this paper is set up as follows: in section 2, we briefly describe IEL and action models. Then we refine the definition of an action model to work with inquisitive epistemic models in section 3. We will show that this is not yet enough, and that we have to make action models inquisitive as well, which we will do in section 4. In section 5 we prove that our new inquisitive action models can deal with everything that IDEL, AML and PAL can deal with. The last section contains the conclusion and some suggestions for further work.

# 2 Current work

## 2.1 Inquisitive Epistemic Logic

In inquisitive logic, propositions are identified with downward closed sets of information states rather then sets of worlds, to encode both informative and inquisitive content. As a result, inquisitive logics can express not just statements, but also questions.[2]

Models of inquisitive epistemic logic encode not only the knowledge agents have, but also their issues or epistemic goals: the things they want to know. In IEL we can express propositions about the knowledge of agents, but also about the issues they entertain.

Definition 2.1. **Inquisitive Epistemic Model** (Ciardelli & Roelofsen, 2015)
An inquisitive epistemic model is a triple $M = \langle W, \{\Sigma_a \mid a \in \mathcal{A}\}, V\rangle$ where

- $W$ is the domain of worlds;
- $\mathcal{A}$ is the domain of agents;
- $\Sigma_a$ is a state map for agent $a$ such that for each world $w$, $\Sigma_a(w)$ is the inquisive proposition that encodes $a$'s knowledge state and goals at $w$. $\Sigma_a(w)$ is a non-empty downward closed set of information states. The knowledge state of $a$ is $\sigma_a(w) = \bigcup \Sigma_a(w)$: the set of worlds she considers possible at $w$. Her goal is to get her knowledge state to be one of the information states in $\Sigma_a(w)$;
- $V$ is a valuation function.

Because we are dealing with espistemic logic, we require the state map to satisfy factivity (for all $w \in W$, $w \in \sigma_a(w)$) and introspection (for all $w, v \in W$, if $v \in \sigma_a(w)$ then $\Sigma_a(v) = \Sigma_a(w)$).

[2] We refer the reader who is not yet familiar with the notions of inquisitive logic to Ciardelli (2016), as it requires more introduction then we can cover here.

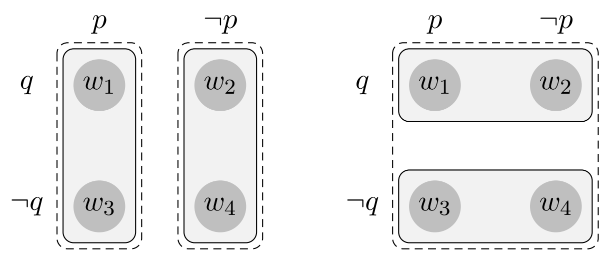
Example 2.1. **Inquisitive Epistemic Model**
As an example, take the following model $M$ with $W = \{w_1, w_2, w_3, w_4\}$, $V(p) = \{w_1, w_3\}$, $V(q) = \{w_1, w_2\}$ and two agents $a$ and $b$. Suppose $a$ knows whether $p$ and not whether $q$, but does not care about it. $b$ does not know whether $p$ or whether $q$, but wants to know whether $q$. The model will look like this:

$\Sigma_a(w_1) = \Sigma_a(w_3) = \{\{w_1, w_3\}\}^{\downarrow}$[3]

$\Sigma_a(w_2) = \Sigma_a(w_4) = \{\{w_2, w_4\}\}^{\downarrow}$

$\Sigma_b(w_1) = \Sigma_b(w_2) = \Sigma_b(w_3) = \Sigma_b(w_4) = \{\{w_1, w_2\}, \{w_3, w_4\}\}^{\downarrow}$

For clarity, we will represent inquisitive epistemic models by diagrams. We follow the convention of Ciardelli (2016): for each world $w$, the worlds within the same dashed line are the accessible worlds: the worlds held possible in $w$. The solid lines represent the issues within each epistemic state: these states and their subsets are the ones that the agent strives to be in. The example model $M$ is represented in Figure 1.




(a) State map for $a$ (b) State map for $b$

Figure 1: Example of state maps in an inquisitive epistemic model

As we will use the language $\mathcal{L}^{\mathsf{IEL}}$ to describe our models and action content, we now briefly introduce its syntax and semantics here.

Definition 2.2. **Syntax of $\mathcal{L}^{\mathsf{IEL}}$** (Ciardelli, 2016)

$$\varphi ::= p \mid \bot \mid \varphi \wedge \varphi \mid \varphi \to \varphi \mid \varphi \mathbin{\vee\!\!\!\vee} \varphi \mid K_a\varphi \mid E_a\varphi$$

For now, we leave out the common knowledge modality, because we will not use it in this paper. We do use the following abbreviations:

| | |
|---|---|
| Negation: | $\neg\varphi := \varphi \to \bot$ |
| Classic disjunction: | $\varphi \vee \psi := \neg(\neg\varphi \wedge \neg\psi)$ |
| Wonder modality: | $W\varphi := \neg K\varphi \wedge E\varphi$ |
| Question operator: | $?\varphi := \varphi \mathbin{\vee\!\!\!\vee} \neg\varphi$ |

The semantics of inquisitive logics are usually not given in terms of truth in a world, but in terms of support in an information state. Truth conditions are then defined indirectly.

[3]The downward closure $\{\{w_1, w_3\}\}^{\downarrow}$ of $\{\{w_1, w_3\}\}$ is $\{\{w_1, w_3\}, \{w_1\}, \{w_3\}, \emptyset\}$.

Definition 2.3. **Semantics of IEL** (Ciardelli, 2016)
Let $M$ be an inquisitive epistemic model, $s$ an information state and $w$ a world.

$$
\begin{array}{lll}
M, s \models p & \text{iff} & V(w, p) = 1 \text{ for all } w \in s \\
M, s \models \bot & \text{iff} & s = \emptyset \\
M, s \models \varphi \wedge \psi & \text{iff} & s \models \varphi \text{ and } s \models \psi \\
M, s \models \varphi \rightarrow \psi & \text{iff} & \text{for all } t \subseteq s,\ t \models \varphi \text{ implies } t \models \psi \\
M, s \models \varphi \vee\!\!\!\vee \psi & \text{iff} & s \models \varphi \text{ or } s \models \psi \\
M, s \models K\varphi & \text{iff} & \text{for all } w \in s : M, \sigma(w) \models \varphi \\
M, s \models E\varphi & \text{iff} & \text{for all } w \in s, \text{ for all } t \in \Sigma(w) : M, t \models \varphi \\
M, w \models \varphi & \text{iff} & M, \{w\} \models \varphi
\end{array}
$$

The modalities $K$ and $E$ are the same when combined with a statement, in which case they express knowledge of that statement. Combined with an inquisitive formula however, they are different: $K_a?\varphi$ means $a$ knows $?\varphi$ (she either knows that $\varphi$ or that $\neg\varphi$), while $E_a?\varphi$ means $a$ *entertains* $?\varphi$: she either knows or wants to know whether $\varphi$. The wonder modality $W$ abbreviates the combination of not knowing and wanting to know.

Note also the difference between a classical disjunction ($p \vee q$) and an inquisitive disjunction ($p \vee\!\!\!\vee q$): both formulas express the information that $p$ or $q$ is true, but the inquisitive disjunction also raises the issue whether it is $p$ or $q$, which is resolved by finding out that $p$ or finding out that $q$.

## 2.2 Action Models

Consider two agents $a$ and $b$ who flip a coin, after which only $a$ sees the outcome. After this event, $a$ can distinguish between the worlds in which it came out heads and the worlds in which it came out tails, while $b$ cannot. In addition, it is common knowledge that $a$ knows the outcome and $b$ does not.

In dynamic epistemic logic, epistemic actions which are more complex than public announcements can be modeled using action models. These models encode the knowledge that agents have about the action taking place, which can differ from agent to agent. The action model is a Kripke model containing all possible actions (in the example, $\mathsf{heads}$ and $\mathsf{tails}$), an accessibility relation for each agent (which encodes their knowledge about the action taking place) and for each action a precondition: a logical formula that encodes what must be true for the action to be possible. An introduction to the notions of action models can be found in Van Ditmarsch et al. (2007, chapter 6).

Definition 2.4. **Action Model** (Van Ditmarsch et al., 2007)
An action model is a triple $\mathsf{M} = \langle \mathsf{S}, \{\sim_a \mid a \in \mathcal{A}\}, \mathsf{pre} \rangle$ where

- $\mathsf{S}$ is a domain of action points;
- For each $a \in \mathcal{A}$, $\sim_a$ is an equivalence relation on $\mathsf{S}$;
- $\mathsf{pre} : \mathsf{S} \rightarrow \mathcal{L}$ is a function that assigns a content $\mathsf{pre}(\mathsf{x}) \in \mathcal{L}$ to each action point $\mathsf{x} \in \mathsf{S}$.

The original epistemic model and the action model are combined to create a new epistemic model: a restricted modal product of the two. This new model encodes the knowledge of the agents after the action has taken place.

Definition 2.5. **Updated model** (Van Ditmarsch et al., 2007)
$M' = (M \otimes \mathsf{M})$ is a restricted modal product of an epistemic model $M$ and an action model $\mathsf{M}$.

$M' = \langle W', \{\sim'_a \mid a \in \mathcal{A}\}, V' \rangle$, where:

- $W' = \{(w, \mathsf{x}) \mid w \in W, \mathsf{x} \in \mathsf{S} \text{ and } M, w \models \mathsf{pre}(\mathsf{x})\}$
- $(w, \mathsf{x}) \sim'_a (w', \mathsf{x}')$ iff $w \sim_a w'$ and $\mathsf{x} \sim_a \mathsf{x}'$
- $(w, \mathsf{x}) \in V'(p)$ iff $w \in V(p)$

So far, Action Model Logic has been an extension of non-inquisitive epistemic logic. Therefore, its models encode only information, not issues. The update procedure can only deal with actions that are based on statements, not questions. In the next section we will see that we can change the definition of action models in a straightforward way to lift it to the inquisitive setting.

### 2.3 Dynamic logics of questions

Before we continue, we briefly mention some related work in dynamic epistemic logic in which questions play a role. Baltag (2001) develops a dynamic logic in which dialogue games with several types of communication acts, including questions, can be described. However, the approach most similar to ours is DELQ (Van Benthem & Minică, 2012), which also has an update procedure for information and issues. The most important difference is that in these systems, issues are encoded in partitions, which means resolutions to issues are mutually exclusive and cannot overlap, in contrast to issues in the inquisitive setting. We will return to this in Section 6.

## 3 Action Models for IEL

### 3.1 Defining the Action Model

The most important difference between standard and inquisitive epistemic models is that, while standard epistemic models associate each world $w$ with a set of accessible worlds $\sigma(w)$, inquisitive epistemic models associate each world $w$ with an inquisitive proposition $\Sigma(w)$. This is not a set of worlds, but a set of sets of worlds, which encodes issues (each set of worlds is an information state that resolves the issues of the agent) and knowledge (the set of accessible worlds $\sigma(w) = \bigcup \Sigma(w)$).

A first step to action models for IEL is therefore to make them compatible with the inquisitive setting. In this step, we can leave the definition almost the same as Definition 2.4.

Definition 3.1. **Action Model for IEL**
An action model for IEL is a triple $\mathsf{M} = \langle \mathsf{S}, \{\sim_a \mid a \in \mathcal{A}\}, \mathsf{cont} \rangle$, where:

- $\mathsf{S}$ is a domain of action points;
- For each $a \in \mathcal{A}$, $\sim_a$ is an equivalence relation on $\mathsf{S}$;
- $\mathsf{cont} : \mathsf{S} \to \mathcal{L}^{\mathsf{IEL}}$ is a function that assigns a content $\mathsf{cont}(\mathsf{x}) \in \mathcal{L}^{\mathsf{IEL}}$ to each action point $\mathsf{x} \in \mathsf{S}$.

This definition replaces $\mathsf{pre}$ with $\mathsf{cont}$. The function $\mathsf{cont}$ maps each action to the *content* of the action rather than the precondition. Although the informative content is still a precondition for an action, in the inquisitive setting there is also inquisitive content, which does more work. For instance, a question like $?p$ has an effect on the issues of the agent that hears it: namely, if she doesn't know the answer, she will come to enterain it. Therefore it is not very intuitive to call $?p$ a precondition.

We now also lift the definition of the updated model to the inquisitive setting.

The domain of our updated model should be a part of $W \times \mathsf{S}$, but, just like in the standard updated model, we want to combine a world with an action only if the world is consistent with the content of that action.

The state maps should be defined in a way that makes sure that (i) it reflects the accessibility relation in the action model: an agent that cannot distinguish between two worlds $w$ and $w'$ and two actions $\mathsf{x}$ and $\mathsf{x}'$ cannot distinguish worlds $(w, \mathsf{x})$ and $(w', \mathsf{x}')$; (ii) issues that the agent has in the old model, should persist in the new model, unless resolved; (iii) issues raised by the content of the action should be reflected in the new model.

For our definition we make use of the projection operator as defined in Definition 3.2.

DEFINITION 3.2. **Projection operator**
$\pi_1(s) := \{w \mid (w, \mathsf{x}) \in s \text{ for some } \mathsf{x}\}$
$\pi_2(s) := \{\mathsf{x} \mid (w, \mathsf{x}) \in s \text{ for some } w\}$

This is useful because it allows us to refer to a state in the original model that is associated with a state in the new model. For example, $\pi_1(\{(w_1, \mathsf{x}), (w_1, \mathsf{y}), (w_2, \mathsf{x})\}) = \{w_1, w_2\}$.

DEFINITION 3.3. **Updated $\mathsf{IEL}$ model**
$M' = (M \otimes \mathsf{M})$ is a restricted modal product of an inquisitive epistemic model $M$ and an action model $\mathsf{M}$.

$M' = \langle W', \{\Sigma'_a \mid a \in \mathcal{A}\}, V' \rangle$, where:

- $W' = \{(w, \mathsf{x}) \mid w \in W, \mathsf{x} \in \mathsf{S} \text{ and } M, w \models \mathsf{cont}(\mathsf{x})\}$
- $t \in \Sigma'_a((w, \mathsf{x}))$ iff
  (i) $\forall (w', \mathsf{x}') \in t, \mathsf{x} \sim_a \mathsf{x}'$
  (ii) $\pi_1(t) \in \Sigma_a(w)$
  (iii) $M, \pi_1(t) \models \mathsf{cont}(\mathsf{x}') \text{ for some } \mathsf{x}' \sim_a \mathsf{x}$
- $(w, \mathsf{x}) \in V'(p)$ iff $w \in V(p)$

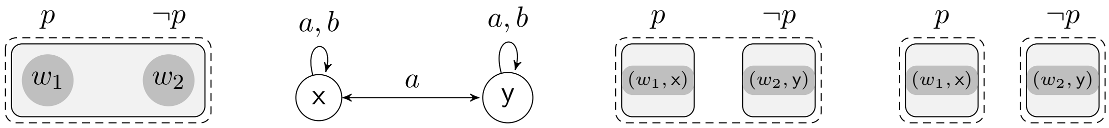
Note that the definition of the domain is essentially the same as in Definition 2.5: the content serves as a precondition for the world to be in the new model. Because a question does not contain information, in the inquisitive setting, questions are trivially true in single worlds. This means that requiring that $M, w \models \mathsf{cont}(\mathsf{x})$ means requiring that the *informative* content of $\mathsf{x}$ is true in $w$, while the inquisitive content does not have any effect here.

The definition of the state map is motivated as follows: condition (i) makes sure that only tuples of which the action was accessible in the action model, will be accessible in the new model. Condition (ii) makes sure that the accessibility relation and issues from the old model are reflected in the new model. Finally, condition (iii) takes care of new issues that should be raised in case the content of a certain action was inquisitive, by requiring states to resolve the content of some accessible action.

## 3.2 Testing the Action Model

Let us now look at three examples to see if our model works the way we expect it too.

EXAMPLE 3.1. **Two statements**
Let the initial state maps for agents $a$ and $b$ and the action model be defined as in Figure 2. Let $\mathsf{cont}(\mathsf{x}) = p$ and $\mathsf{cont}(\mathsf{y}) = \neg p$. The results are shown in the same figure.




(a) State map of $a$ and $b$ before action

(b) Action model

(c) State map of $a$ after action

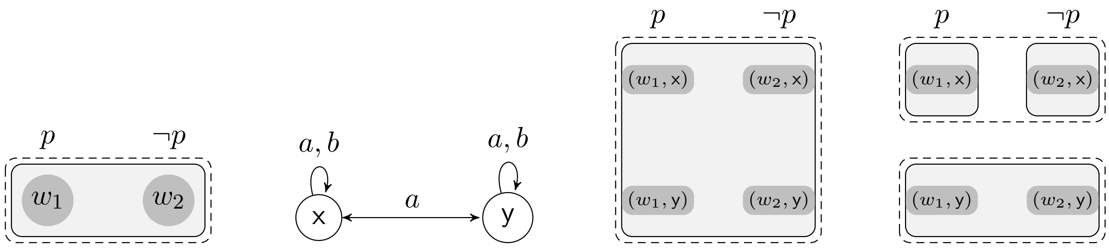
(d) State map of $b$ after action

Figure 2: Example 3.1

According to the action model, $a$ does not know what action is happening. She does not learn anything, which is reflected in her model. What changes, however, is that it becomes an issue for her whether $p$, because we require states to support the content of some action. In contrast, $b$ now knows whether $p$, and this is common knowledge. What this example shows is that using the above recipe, agents are automatically interested in what takes place.

EXAMPLE 3.2. **A question or nothing**
Let the initial state maps for agents $a$ and $b$ and the action model be defined as in Figure 3. This time, let $\mathsf{cont}(\mathsf{x}) = ?p$ and $\mathsf{cont}(\mathsf{y}) = \top$. The results are shown in the same figure.

The resulting model for $b$ reflects exactly what is happening: either she now entertains whether $p$ (in case the action was $\mathsf{x}$), or she does not ($\mathsf{y}$), and this is common knowledge. In contrast, $a$ does not get any new issues at all, as it could be the case that $\top$ was the content of the action. This is a somewhat unintuitive result when we compare it to Example 3.1, in which $a$ also did not know what happened, but automatically developed the issue whether $p$.




(a) State map of $a$ and $b$ before action

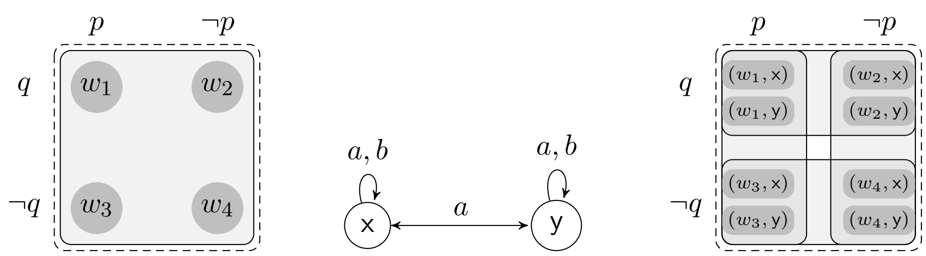
(b) Action model

(c) State map of $a$ after action

(d) State map of $b$ after action

Figure 3: Example 3.2

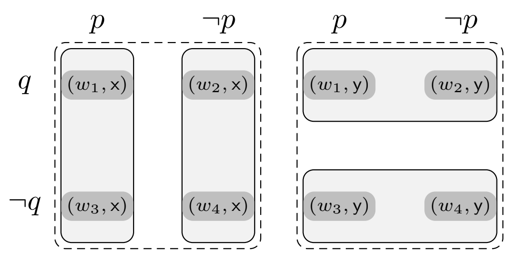
Example 3.3. **Two questions**
Let us now consider an action model with two questions: $\mathsf{cont}(\mathsf{x}) = ?p$ and $\mathsf{cont}(\mathsf{y}) = ?q$. The initial state map, action model and results are shown in Figure 4.




(a) State map of $a$ and $b$ before action

(b) Action model

(c) State map of $a$ after action




(d) State map of $b$ after action

Figure 4: Example 3.3

Again, the result for $b$ is exactly what we would expect: depending on the action that actually took place, either she entertains whether $p$ or she entertains whether $q$. On the other hand, $a$ cannot distinguish whether one or the other is the case. Although it is not an issue for $a$ whether $\mathsf{x}$ or $\mathsf{y}$ happened, she does entertain $?p \vee\!\!\!\vee\, ?q$.

What we can conclude from these examples is that our current action model and update procedure make the correct predictions for agents who have knowledge about the action happening. However, the predictions for agents who do not have full knowledge are not yet exactly as we want them, because we do not have enough control over which issues they develop in the resulting model. We need to be able to define issues over action models, which we can achieve by lifting the action model to the inquisitive setting as well. The next section is dedicated to developing this idea.

# 4 Making Action Models inquisitive

## 4.1 Defining the Inquisitive Action Model

In the previous section we saw some unexpected results with respect to issues in the updated model. We suggest that this problem is caused by the fact that the new action model is still too much like the standard action model: it expresses only knowledge about the action that is taking place, not whether this is actually an issue for the agent.

A natural solution to this problem would be to go one step further, by also lifting the action model itself to the inquisitive setting. We do this by giving the accessibility relation of the action model more structure, analogous to the way inquisitive epistemic logic went from knowledge maps to state maps. Instead of defining an accessibility relation $\sim$, we define a function $\Delta$, which maps each action point to a non-empty downward closed set of action points. This set represents not only which actions are accessible, but also groups them together into states that resolve the agents issues as to which action is taking place.

Definition 4.1. **Inquisitive Action Model**
An Inquisitive Action Model for $\mathsf{IEL}$ is a triple $\mathsf{M} = \langle \mathsf{S}, \{\Delta_a \mid a \in \mathcal{A}\}, \mathsf{cont}\rangle$, where:

- $\mathsf{S}$ and $\mathsf{cont}$ are defined as in Definition 3.1
- For each $a \in \mathcal{A}$, $\Delta_a$ is a function that maps an action point to a non-empty downward closed set of sets of action points.

Recall that in our epistemic model, the information state $\sigma_a(w)$ is defined as the union of the inquisitive proposition $\Sigma_a(w)$. This is simply the set of worlds that are accessible from $w$, which represents the information at $w$. Similarly, in Inquisitive Action Models, we define $\delta_a(\mathsf{x}) := \bigcup \Delta_a(\mathsf{x})$ for each action $\mathsf{x}$. This means that $\delta_a(\mathsf{x})$ is the set of actions indistinguishable from $\mathsf{x}$ by agent $a$.

Note that the accessibility relation $\sim_a$ is therefore still encoded in the system, because it is completely determined by $\Delta_a$:

$\sim_a := \{\langle \mathsf{x}, \mathsf{x}' \rangle \mid \mathsf{x}' \in \delta_a(\mathsf{x})\}$

As we are still dealing with epistemic logic, we require that $\sim$ is an equivalence relation. This means we require that $\Delta$ satisfies the same factivity and introspection conditions as $\Sigma$.

Example 4.1. **Inquisitive Action Model**
Consider the two actions $\mathsf{x}$ and $\mathsf{y}$. If agent $a$ knows which action is the actual one, $\Delta_a(\mathsf{x}) = \{\{\mathsf{x}\}\}^{\downarrow}$ and $\Delta_a(\mathsf{y}) = \{\{\mathsf{y}\}\}^{\downarrow}$. If agent $b$ does not know which action is the actual one, but *wants* to know, $\Delta_b(\mathsf{x}) = \Delta_b(\mathsf{y}) = \{\{\mathsf{x}\}, \{\mathsf{y}\}\}^{\downarrow}$. If agent $c$ does not know which action is the actual one, and does not care either, then $\Delta_c(\mathsf{x}) = \Delta_c(\mathsf{y}) = \{\{\mathsf{x}, \mathsf{y}\}\}^{\downarrow}$.

We can now change our definition of an updated model to work with inquisitive action models, in particular the definition of an updated state map. We should make sure that each updated state map (i) still reflects the accessibility relation in the inquisitive action model: an agent that cannot distinguish between two worlds $w$ and $w'$ and between two actions $\mathsf{x}$ and $\mathsf{x}'$ cannot distinguish worlds $(w, \mathsf{x})$ and $(w', \mathsf{x}')$, but on top of that, it should be reflected to what extent the agent considers it an issue which action has happened; (ii) as in the previous definition, inherits the issues from the old model unless resolved; (iii) issues raised by the content of the action should be reflected in the new model, but now we can group issues together that belong to actions of which it is not an issue whether one or the other has taken place.

Definition 4.2. **IEL model updated with an Inquisitive Action Model**
$M' = (M \otimes \mathsf{M})$ is a restricted modal product of an inquisitive epistemic model $M$ and an inquisitive action model $\mathsf{M}$.

$M' = \langle W', \{\Sigma'_a \mid a \in \mathcal{A}\}, V' \rangle$, where:

- $W'$ and $V'$ are defined as in Definition 3.3.
- $t \in \Sigma'_a((w, \mathsf{x}))$ iff
  - (i) $\pi_2(t) \in \Delta_a(\mathsf{x})$
  - (ii) $\pi_1(t) \in \Sigma_a(w)$
  - (iii) There is some non-empty $\mathsf{s} \in \Delta_a(\mathsf{x})$ such that $\pi_2(t) \subseteq \mathsf{s}$ and $M, \pi_1(t) \models \bigvee\{\mathsf{cont}(\mathsf{x}') \mid \mathsf{x}' \in \mathsf{s}\}$[4]

Condition (i) does the same work as in Definition 3.3, but now it als keeps the structure of issues from the action model, similar to condition (ii), which is unchanged. Condition (iii) is motivated by the following observations. To reflect new issues raised by the actions in the updated model, we need to require new states to resolve these issues. However, not all states have to resolve all the issues raised by the content of all actions: for example, states that contain only $\mathsf{x}$-worlds have to resolve only the issues raised by the content of $\mathsf{x}$. This keeps issues nicely separated: in $\mathsf{x}$-worlds, agents consider the issues raised by the content of $\mathsf{x}$ (if any), and in $\mathsf{y}$-worlds, agents consider the issues raised by the content of $\mathsf{y}$. For agents who know the actual action, only the issue raised by the content of the actual action will become an issue. For agents who cannot distinguish between $\mathsf{x}$-worlds and $\mathsf{y}$-worlds, both issues play a role, in a way we will see in the next section.

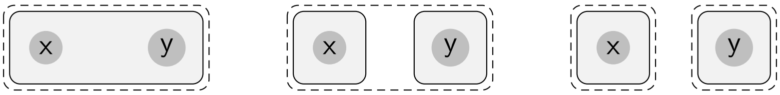
Things are different for an agent who does not care: say there is an agent $a$ that does not care whether $\mathsf{x}$ or $\mathsf{y}$ happened, and we have $\mathsf{cont}(\mathsf{x}) = p$ and $\mathsf{cont}(\mathsf{y}) = q$. The desired outcome would then be that she now considers only worlds where $p$ or $q$ is true, but it is not an issue for her whether one or the other is the case. That means we have to require $p \vee q$ to be supported by her states, not $p \veebar q$, because in the latter case it would automatically become an issue whether $p$ or $q$.

Being able to "group" action content, without raising issues between it, motivates the use of a classical disjunction in our definition. As an effect, if there are two actions with inquisitive content, like $?p$ and $?q$, no issues at all will be raised for the agent who does not care (because $?p \vee ?q \equiv \top$, which is supported by all states).

[4]Where $\bigvee \Phi$ should be interpreted as the *classic, non-inquisitive* disjunction of all formulas in the set $\Phi$ in case it contains more than one formula, and just the single formula otherwise.

### 4.2 Testing the Inquisitive Action Model

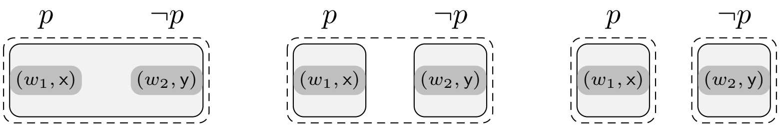
EXAMPLE 4.2. **Two statements**
We repeat Example 3.1, but now with 3 agents: $a$, $b$ and $c$. Only $c$ knows which action is happening. The difference between $a$ and $b$ is that only $b$ cares about which action is happening. No agent has any knowledge or issue in the original state map. The state maps of the action model are drawn in Figure 5 and now look exactly like state maps from an inquisitive epistemic model.




(a) State map for $a$ (b) State map for $b$ (c) State map for $c$

Figure 5: Example 4.2: inquisitive action model

The updated models are shown in Figure 6 and reflect the issues from the action models.



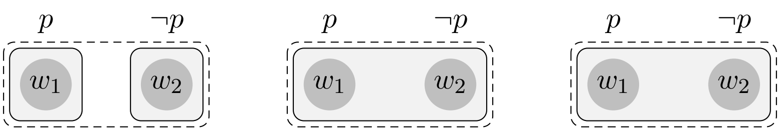

(a) State map for $a$ (b) State map for $b$ (c) State map for $c$

Figure 6: Example 4.2: state maps after action

This example shows that in this setting, an agent who does not know which action has happened does not necessarily develop an issue concerning the content of the actions: this now depends on whether it was an issue for her which action occurred.

Let us also repeat Example 3.2, but with different agents.

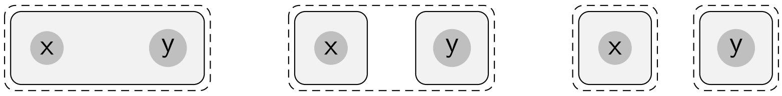
EXAMPLE 4.3. **A question or nothing**
Let $\mathsf{cont}(\mathsf{x}) = ?p$ and $\mathsf{cont}(\mathsf{y}) = \top$. The state maps of the original model are shown in Figure 7: no one knows whether $p$, but $a$ entertains $?p$: it is her goal to find out if $p$ is true or false.




(a) State map for $a$ (b) State map for $b$ (c) State map for $c$

Figure 7: Example 4.3: state maps in the original model

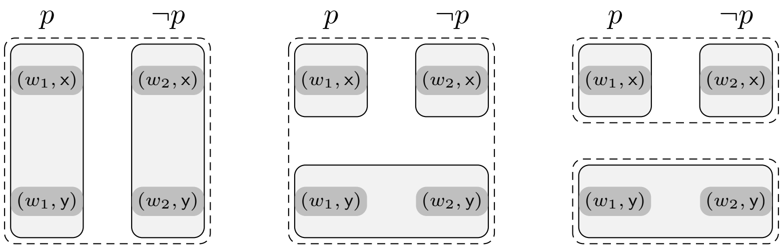
The action models are represented in Figure 8: $a$ neither knows nor cares what is happening; $b$ also does not know, but does care; $c$ knows what is happening.




(a) State map for $a$ (b) State map for $b$ (c) State map for $c$

Figure 8: Example 4.3: inquisitive action model

The resulting models are shown in Figure 9. Because $a$ did not care about the action, her updated model does not show a new issue. However, it does show her old issue whether $p$, which she already had in the original model. For $b$ it has become an issue whether action $\mathsf{x}$ or $\mathsf{y}$ happened. It is not an issue whether $p$ (as there is one alternative in which $?p$ is not resolved), but if the worlds where action $\mathsf{y}$ happened were dropped, this would become an issue, which seems intuitive.




(a) State map for $a$ (b) State map for $b$ (c) State map for $c$

Figure 9: Example 4.3: state maps in the updated model

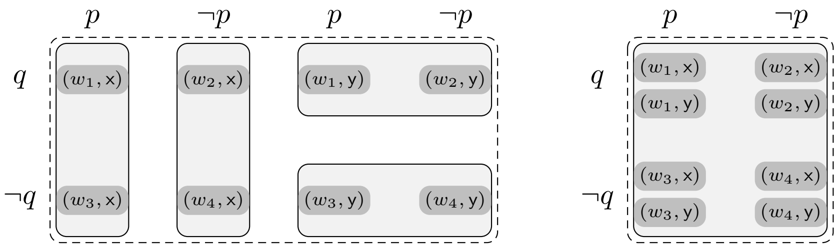
What is interesting about the state map of $b$ is that the outcome is intuitive, but the issue whether one or the other action happened is not expressible in the language, as there is no propositional difference between worlds $(w_1, \mathsf{x})$ and $(w_1, \mathsf{y})$. However, if $b$ would come to know that $c$ is entertaining $?p$, she could drop the worlds in which this is not the case (the $\mathsf{y}$-worlds) and she would automatically start entertaining $?p$ herself.

Let us also look at what comes out if we try what we did in Example 3.3 again.

Example 4.4. **Two questions**
Let $\mathsf{cont}(\mathsf{x}) = ?p$ and $\mathsf{cont}(\mathsf{y}) = ?q$. Both $a$ and $b$ have no knowledge or issues in the original model, and no knowledge about which action is happening. The only difference is that $a$ cares about the action and $b$ does not care. The result is shown in Figure 10.

Observe that the resulting model makes the following sentences true:

- $\neg K_a ?p$, $\neg K_a ?q$, $\neg K_b ?p$, $\neg K_b ?q$: $a$ and $b$ do not know whether $p$ and not whether $q$.
- $\neg W_a ?p$, $\neg W_a ?q$, $\neg W_b ?p$, $\neg W_b ?q$: $a$ and $b$ do not wonder whether $p$ and not whether $q$.
- $W_a(?p \vee\!\!\!\vee ?q)$: $a$ wonders whether $?p$ or $?q$.




(a) State map of $a$ after action

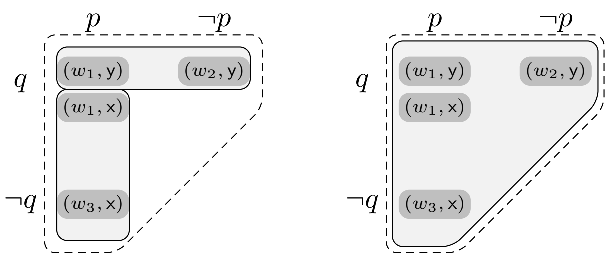
(b) State map of $b$ after action

Figure 10: Example 4.4

- $K_b W_a(?p \vee\!\!\!\vee ?q)$: $b$ knows this.

If an agent does not consider it an issue which action is taking place, she will not have any new issues in the updated model. However, she can still have new knowledge, as we can see in the following example.

Example 4.5. **Two statements**
Let $\mathsf{cont}(\mathsf{x}) = p$ and $\mathsf{cont}(\mathsf{y}) = q$. Both $a$ and $b$ have no knowledge or issues in the original model, and no knowledge about which action is happening. $a$ cares about the action and $b$ does not care. The result is shown in Figure 11.




(a) State map of $a$ after action

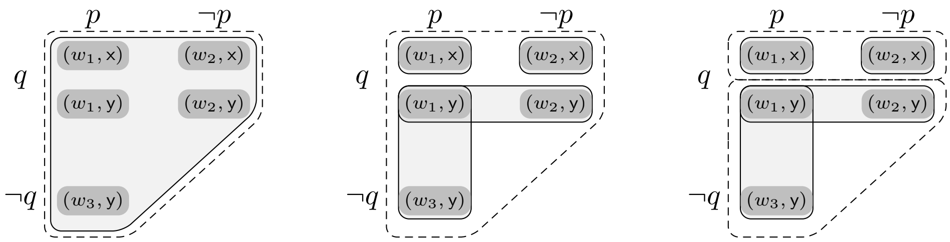
(b) State map of $b$ after action

Figure 11: Example 4.5

As both agents know that the action content was either $p$ or $q$, they now both know that $p \vee q$ is true. However, agent $a$ also entertains $p \vee\!\!\!\vee q$, while $b$ does not.

It is also worth noting that updates with hybrid formula's work as expected in this system. Hybrids contain informative as well as inquisitive content.

Example 4.6. **Hybrids**
Let $\mathsf{cont}(\mathsf{x}) = ?p \wedge q$ and $\mathsf{cont}(\mathsf{y}) = p \vee\!\!\!\vee q$. Both contents are hybrid: the content of $\mathsf{x}$ contains the information that $q$ and the issue whether $p$, the content of $\mathsf{y}$ contains the information that $p \vee q$ and the issue whether $p$ or $q$. As in the previous example, all agents are completely ignorant and without issues in the original model. This time, let $c$ know which action is the actual one, let $b$ not know but care, and let $a$ not care (the action model is the same as in Figure 8). The resulting model is shown in Figure 12.




(a) State map of $a$ after action

(b) State map of $b$ after action

(c) State map of $c$ after action

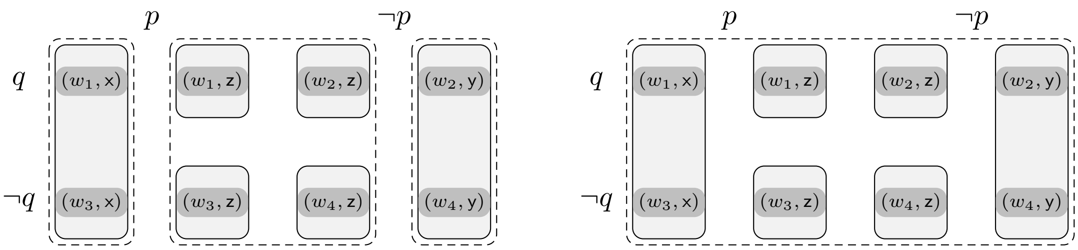
Figure 12: Example 4.6

The result is that all agents learn that it is not the case that $\neg p \wedge \neg q$. For $a$, this is all that happens. Agent $b$ now entertains the complex issue $(?p \wedge q) \vee\!\!\vee (p \vee\!\!\vee q)$, while agent $c$ either learns that $q$ and entertains whether $p$, or entertains whether $p$ or $q$, depending on which action is the actual one.

Note that $b$'s issue can only be settled by learning that $p \wedge q$, $\neg p$ or $\neg q$. However, in case the actual world is a $w_1$-world, $b$ can never come to know whether it is $(w_1, \mathsf{x})$ or $(w_1, \mathsf{y})$, as the difference between these worlds is not expressible in our language.

As a final example, we will show that we can now represent the situation we described in Section 1.1.

Example 4.7. **Anne and Bill**
Let us assume that in the original model, both $a$ and $b$ have no knowledge about $p$ or $q$. However, it is for both of them an issue whether $p$. We define an action model with three actions: $\mathsf{x}$, $\mathsf{y}$ and $\mathsf{z}$. Let $\mathsf{cont}(\mathsf{x}) = p$, $\mathsf{cont}(\mathsf{y}) = \neg p$ and $\mathsf{cont}(\mathsf{z}) = ?q$. $a$ knows exactly which action is happening. $b$ does not, but it is an issue for him. The result is shown in Figure 11.




(a) State map of $a$ after action

(b) State map of $b$ after action

Figure 13: Example 4.7

In this model, every state resolves $?p$, as this was already an issue in the previous model. Resolving this is the main goal of both agents. In case the actual world is a $\mathsf{x}$-world or an $\mathsf{y}$-world, $a$ now knows the result, and has no issues any more If the actual world is a $\mathsf{z}$-world, she still has the issue whether $?p$, but now also $?q$.

# 5 Proofs

In this section we provide some proofs about inquisitive action models. We want to show that they are defined in a way that always works: they can never "break" inquisitive epistemic models. We also show that the way we defined inquisitive action models and the resulting models of updating with them, is conservative with respect to $\mathsf{IDEL}$, $\mathsf{PAL}$ and $\mathsf{AML}$. We claim that every update that can be expressed in each of these three systems can also be expressed using an inquisitive action model, and that the resulting models are the same.

Proposition 5.1. **Updates result in inquisitive epistemic models**
For any inquisitive epistemic model $M$ and for any inquisitive action model $\mathsf{M}$, $M' = (M \otimes \mathsf{M})$ is an inquisitive epistemic model.

*Proof:* Take an arbitrary agent $a$ and world $(w, \mathsf{x}) \in W'$.

We will first show that $\Sigma'_a((w, \mathsf{x}))$ is downward closed. Take any state $t \in \Sigma'_a((w, \mathsf{x}))$ and any $u \subseteq t$.

(i) $\pi_2(u) \in \Delta_a(\mathsf{x})$ by $\pi_2(u) \subseteq \pi_2(t)$ and downward closure of $\Delta_a(\mathsf{x})$.

(ii) $\pi_1(u) \in \Sigma_a(w)$ by $\pi_1(u) \subseteq \pi_1(t)$ and downward closure of $\Sigma_a(w)$.

(iii) We know that $t$ satisfies condition (iii), so there is some non-empty $\mathsf{s} \in \Delta_a(\mathsf{x})$ such that $\pi_2(t) \subseteq \mathsf{s}$ and $M, \pi_1(t) \models \bigvee\{\mathsf{cont}(\mathsf{x}') \mid \mathsf{x}' \in \mathsf{s}\}$. This $\mathsf{s}$ can be reused for $u$, as $\pi_2(u) \subseteq \pi_2(t) \subseteq \mathsf{s}$. As $\pi_1(u) \subseteq \pi_1(t)$, it is a more specific information state than $\pi_1(t)$. Therefore it supports all formulas that $\pi_1(t)$ supports.

This concludes downward closure. Next, to show factivity (and non-emptiness), consider the state $\{(w, \mathsf{x})\}$.

(i) $\{\mathsf{x}\} \in \Delta_a(\mathsf{x})$ by factivity and downward closure of $\Delta_a(\mathsf{x})$.

(ii) $\{w\} \in \Sigma_a(w)$ by factivity and downward closure of $\Sigma_a(w)$.

(iii) $M, \{w\} \models \mathsf{cont}(\mathsf{x})$ by definition of $W'$.

That leaves introspection. Take any two worlds $(w, \mathsf{x})$ and $(w', \mathsf{x}')$ from $W'$ such that $(w', \mathsf{x}') \in \sigma'_a((w, \mathsf{x}))$. By downward closure of $\Sigma'_a((w, \mathsf{x}))$ we obtain $\{(w', \mathsf{x}')\} \in \Sigma'_a((w, \mathsf{x}))$. From condition (i) we learn that it must be the case that $\mathsf{x}' \in \bigcup \Delta_a(\mathsf{x})$. By introspection of $\Delta_a$ we obtain $\Delta_a(\mathsf{x}) = \Delta_a(\mathsf{x}')$. Via condition (ii) we can obtain in a similar way that $\Sigma_a(w) = \Sigma_a(w')$. Then it is easy to check that for all states $t$, $t$ satisfies conditions (i-iii) for $\Sigma'_a((w, \mathsf{x}))$ iff it satisfies them for $\Sigma'_a((w', \mathsf{x}'))$. Therefore $\Sigma'_a((w, \mathsf{x}))$ and $\Sigma'_a((w', \mathsf{x}'))$ are equal.

□

Proposition 5.2. **Equivalence with public utterance in IDEL**
For any inquisitive epistemic model $M$ and for any formula $\varphi \in \mathcal{L}^{\mathsf{IEL}}$, the model $M^{\varphi}$ that is the result of a public utterance of $\varphi$ in $\mathsf{IDEL}$ is equivalent to $M' = M \otimes \mathsf{M}$ where $\mathsf{M} = \langle\{\mathsf{pub}\}, \{\Delta_a \mid a \in \mathcal{A}\}, \mathsf{cont}\rangle$, $\Delta_a(\mathsf{pub}) = \{\{\mathsf{pub}\}\}^{\downarrow}$ for each $a \in \mathcal{A}$ and $\mathsf{cont}(\mathsf{pub}) = \varphi$.[5]

[5] For the definition of an update with a public utterance in IDEL, see Definition 8.2.1 in Ciardelli (2016).

*Proof:* As there is only one action $\mathsf{pub}$ in $\mathsf{M}$, for all worlds $w \in W$ we can use the name $w$ as an abbreviation for $(w, \mathsf{pub}) \in W'$. The definition of $W'$ can then be rewritten as $W' = \{w \in W \mid M, w \models \varphi\}$. This means that $W'$ contains all and only the worlds in $W$ in which (the informative content of) $\varphi$ is true. Therefore $W' = W \cap \mathsf{info}(\varphi)$.

Take an arbitrary agent $a$ and an arbitrary world $w \in W'$ and consider the definition of $\Sigma'_a(w)$. As there is only one action that is accessible to all agents, condition (i) is trivially satisfied by all information states. Condition (ii), using the abbreviated world names, becomes $t \in \Sigma_a(w)$. Condition (iii) can be shortened as $M, t \models \varphi$, which is equivalent to $t \in [\varphi]$. So the states in $\Sigma'_a(w)$ are exactly the states $t$ such that $t \in \Sigma_a(w)$ and $t \in [\varphi]$. So $\Sigma'_a(w) = \Sigma_a(w) \cap [\varphi]$.

The valuation functions are trivially equal.

□

We have shown that a public utterance in $\mathsf{IDEL}$ is equal to a public utterance expressed by an Inquisitive Action Model. It immediately follows from Proposition 5.2 and Proposition 8.2.6 of Ciardelli (2016) that a public utterance of a statement in an Inquisitive Action Model is equal to a public announcement in $\mathsf{PAL}$.

We will now show that Inquisitive Action Models and their update procedure are conservative with respect to standard action models. This requires a comparison between inquisitive and non-inquisitive epistemic models. As non-inquisitive models only encode knowledge and no issues, we can only compare the models in terms of the knowledge they encode. Therefore, we first need to define the notion of model equivalence with respect to knowledge.

Definition 5.1. **Equivalence with respect to knowledge**
Two epistemic models (inquisitive, non-inquisitive or a mix of both) are equivalent with respect to knowledge iff

(i) They share the same domain and valuation

(ii) For each $a \in \mathcal{A}$, their equivalence relations $\sim_a$ and $\sim'_a$ are equivalent. In case a model is inquisitive, let its $\sim_a$ be defined as follows: $w \sim_a w'$ iff $w' \in \delta_a(w)$.

Proposition 5.3. **Equivalence with non-inquisitive action models**
For any non-inquisitive epistemic model $M$ and non-inquisitive action model $\mathsf{M}$, the updated model $M' = M \otimes \mathsf{M}$ is equivalent (with respect to knowledge) to the inquisitive epistemic model $N' = N \otimes \mathsf{N}$ obtained by lifting $M$ to the inquisitive epistemic model $N$ and lifting $\mathsf{M}$ to the inquisitive action model $\mathsf{N}$.

*Proof:* Take any non-inquisitive epistemic model $M = \langle W, \{\sim_a \mid a \in \mathcal{A}\}, V\rangle$ and non-inquisitive action model $\mathsf{M} = \langle \mathsf{S}, \{\sim_a \mid a \in \mathcal{A}\}, \mathsf{pre}\rangle$. Let $N = \langle W, \{\Sigma_a \mid a \in \mathcal{A}\}, V\rangle$ be an inquisitive epistemic model such that for each $w \in W$, $\Sigma_a(w) = \wp(\{w' \mid w \sim_a w'\})$. Similarly, let $\mathsf{N} = \langle \mathsf{S}, \{\Delta_a \mid a \in \mathcal{A}\}, \mathsf{cont}\rangle$ be an inquisitive action model such that for each $\mathsf{x} \in \mathsf{S}$, $\Delta_a(w) = \wp(\{s' \mid s \sim_a s'\})$ and $\mathsf{cont} = \mathsf{pre}$. Then $N$ and $\mathsf{N}$ are the inquisitive counterparts of $M$ and $\mathsf{M}$ respectively: they encode the same knowledge, and no non-trivial issues.

We need to show that $M' = \langle W', \{\sim'_a \mid a \in \mathcal{A}\}, V'\rangle$ and $N' = \langle W'', \{\Sigma''_a \mid a \in \mathcal{A}\}, V''\rangle$ are equivalent with respect to knowledge.

Comparing Definition 2.5 and 4.2 it is trivial that $W'$ is equivalent to $W''$ and $V'$ is equivalent to $V''$. We show that $\sim'_a$ and $\Sigma''_a$ encode the same knowledge by showing that $(w, \mathsf{x}) \sim_a (w', \mathsf{x}')$ iff $(w', \mathsf{x}') \in \bigcup \Sigma''_a(w, \mathsf{x})$.

($\Rightarrow$) Suppose $(w, \mathsf{x}) \sim_a (w', \mathsf{x}')$. Then by Definition 2.5, $w \sim_a w'$ and $\mathsf{x} \sim_a \mathsf{x}'$. By the definitions of $\Sigma_a(w)$ and $\Delta_a(w)$, $w \sim_a w'$ implies $\{w'\} \in \Sigma_a(w)$ and $\mathsf{x} \sim_a \mathsf{x}'$ implies $\{\mathsf{x}'\} \in \Delta_a(\mathsf{x})$. This means that by Definition 4.2, the state $\{(w', \mathsf{x}')\}$ satisfies conditions (i) and (ii) to be included in $\Sigma''_a(w)$. It also satisfies condition (iii) because $\{\mathsf{x}'\} \in \Delta_a(\mathsf{x})$ and by definition of $W''$, $M, w' \models \mathsf{cont}(\mathsf{x})$. From $\{(w', \mathsf{x}')\} \in \Sigma''_a(w)$ we obtain $(w', \mathsf{x}') \in \bigcup \Sigma''_a(w)$.

($\Leftarrow$) We can reverse the left-to-right direction: $(w', \mathsf{x}') \in \bigcup \Sigma''_a(w)$ implies $\{(w', \mathsf{x}')\} \in \Sigma''_a(w)$, which means that state $\{(w', \mathsf{x}')\}$ satisfies all conditions of Definition 4.2 to be in $\Sigma''_a(w)$. From conditions (i) and (ii) we can obtain $w \sim_a w'$ and $\mathsf{x} \sim_a \mathsf{x}'$, which by Definition 2.5 imply $(w, \mathsf{x}) \sim_a (w', \mathsf{x}')$.

This concludes the proof that $M'$ and $N'$ are equivalent with respect to knowledge.

$\square$

## 6 Conclusions and further work

In this paper we have seen that the inquisitive logic framework can deal with action models in a straightforward way. They can be merged into a system which can express everything that IDEL and AML can, and more. Inquisitive action models make it possible to encode situations in which agents do not have full knowledge about the epistemic action taking place, and this epistemic action can have informative content, inquisitive content or both. An interesting feature of inquisitive action models is that the way an agent updates her issues depends on her attitude towards the epistemic action: if she doesn't know which action takes place and doesn't care about it, her attitude towards the informative and inquisitive content that are provided by the actions will reflect this in an intuitive way.

There are many natural suggestions for further work. The most urgent one would be to define an Inquisitive Action Model Logic, analogous to Action Model Logic. This would be the language of IEL, extended with infinitely many extra modalities for each possible action we can express in an inquisitive action model. This could be done very much like the way AML is defined in Van Ditmarsch et al. (2007, chapter 6). As both IDEL and AML have been provided with a sound and complete axiomatization, and given that the two systems can be merged together in a relatively natural way, we can expect this to be possible for IAML as well.

As we have seen in Section 2.3, our approach is similar to DELQ, which also defines product updates for models that encode information and issues. In DELQ, issues are encoded as partitions of the logical space, which means they never overlap. Furthermore, issues are not defined per world but globally, which brings even more limitations to what DELQ can express. A comparison of DELQ and IDEL in Ciardelli (2016) shows that the issues that DELQ can express are a strict subset of the ones that IDEL can express. Because we use the same IEL models as the basis of our dynamic system, much of this comparison carries over to our system. However, a complete comparison of the two systems, including a comparison of the resulting models of product updates in DELQ and IEL with inquisitive action models is still needed.

One of the interesting results we found in this paper is that when updating an inquisitive epistemic model with an inquisitive action model, this can result in a model in which there is an issue that cannot be expressed in the language of IEL. This could be solved by adding new proposition letters to every updated model, expressing the action that has just taken place. This would make the language more expressive, and would allow us to more accurately model a situation like this: $a$ reads either $?p$ or $?q$ from a letter; $b$ doesn't know but wants to know; then $b$ discovers that it was in fact $?p$, and she can now drop the worlds in which she wondered whether $q$, which only leaves the worlds in which she wonders whether $p$.

A third direction for further research would be relaxing the constraints on $\Sigma$ and $\Delta$. When factivity of $\Delta$ is replaced with consistency, this results in action models in which some of the agents are completely unaware of actions taking place, without even considering the actual action. This would give us an inquisitive doxastic model after an update, rather than an inquisitive epistemic model, in which $\Sigma$ no longer satisfies factivity. We expect the definitions given in this paper to still work in such cases, but as doxastic variants of IDEL have not been extensively studied yet, there is much to be discovered in this area.

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