Introduction to natural language semantics
Class 12 – Inquisitive semantics

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MCMP — January 22nd 2019

Today

1. Motivation
2. Foundations
3. Logical operations

Next week

1. Some compositional semantics
2. Embedded questions
3. Inquisitive attitudes

Part I

Why an inquisitive semantics?

Truth-conditions

Traditionally, formal semantics builds on a simple conception of what the meaning of a sentence is, inherited from formal logic.

> *To know the meaning of a sentence is to know its truth-conditions. If I say to you*
>
> *(1) There is a bag of potatoes in my pantry*
>
> *you may not know whether what I said is true. What you do know, however, is what the world would have to be like for it to be true. There has to be a bag of potatoes in my pantry. [...] A theory of meaning, then, pairs sentences with their truth-conditions.*

(Heim and Kratzer (1998), a popular textbook in semantics)

From truth-conditions to classical propositions

- In intensional semantics, a model comes with a universe W of possible worlds, which represent a variety of states of affairs.
- At each possible world, a sentence φ may be true or false.
- The truth-conditions of a sentence can then be encapsulated in a single semantic object: the set of worlds where the sentence is true:

$$|\varphi| = \{w \in W \mid \varphi \text{ is true at } w\}$$

- This is called the proposition expressed by φ (in a context).
- This object can be seen as capturing the informative content of φ.

Entailment

The central notion of entailment is characterized as preservation of truth, i.e., inclusion of propositions:

$$\begin{aligned} \varphi \models \psi \quad &\iff \quad \forall w \in W : \varphi \text{ true at } w \; \Rightarrow \; \psi \text{ true at } w \\ &\iff \quad |\varphi| \subseteq |\psi| \end{aligned}$$

Logical operations

Connectives express operations on propositions:

- $|\neg\varphi| = \overline{|\varphi|}$
- $|\varphi \wedge \psi| = |\varphi| \cap |\psi|$
- $|\varphi \vee \psi| = |\varphi| \cup |\psi|$
- ...

So, the choice of a notion of meaning (truth-conditions) also brings along a corresponding notion of entailment and account of logical operations.

- While this perspective on meaning is very fruitful, it is also limited. It is applicable only to a particular class of sentences: statements.

(2) Alice lives in Paris.

- Natural languages do not comprise only statements, but also other kinds of sentences, in particular, questions.

(3) a. Does Alice live in Paris? — polar question
b. Does Alice live in Paris, or in Rome? — alt. question
c. Where does Alice live? — wh-question

- The meaning of a question does not seem to be characterized by truth-conditions.
- Yet, questions are of primary importance in natural language.

Reason 1. Questions play a crucial role in communication

Questions are used to steer a conversation in certain directions, directing it towards specific issues.

(4)
A	What should we do for the weekend?
B	We could go to the Neue Pinakothek.
A	What kind of art to they have?
B	...

Reason 2: questions occur as constituents of statements

The truth-conditions of the statements depend on the meaning of the constituent who will be elected.

(5)
a. I know/wonder/don't care who will be elected.
b. Bob told me who will be elected.
c. At this stage, no one can predict who will be elected.
d. The prospects of the deal depend on who will be elected.

Reason 3: questions affect the interpretation of statements

The truth-conditions of one and the same statement can be different in the context of two different questions.

(6) A: What did you do this morning?
B: I only read the newspaper. $\rightsquigarrow$ B didn't go jogging

(7) A: What did you read this morning?
B: I only read the newspaper. $\not\rightsquigarrow$ B didn't go jogging

- A theory of semantics must account for the semantics of questions, and thus make semantic objects available as meanings for questions.
- We might keep the standard notion of meaning for statements, and develop a separate notion of meaning for questions.
- This has been the road followed by previous theories of questions:
  - Answer-set theory (Karttunen 1977)
  - Partition theory (Groenendijk and Stokhof 1984)
  - Categorial theory (Tichy 1978, Hausser and Zaefferer 1978)
- Inquisitive semantics provides a new notion of question meaning, which has various advantages over its predecessors.
- But it also differs from previous theories in that it aims at a single, uniform notion of meaning for statements and questions.

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- But it also differs from previous theories in that it aims at a single, uniform notion of meaning for statements and questions.
- Why do we need a uniform notion of meaning?

1. Common building blocks

- Declaratives and interrogatives are to a large extent built up from the same lexical, morphological, and intonational elements.
- E.g., the disjunction or is involved not only in forming disjunctive statements, but also in forming disjunctive questions.

(8) a. Alice lives in Paris or in London.
    b. Does Alice live in Paris, or in London?

- If the or in (8-a) is taken to yield the union of two propositions, then the or in (8-b) needs to be given a different analysis.
- We would like a single, general account of the semantics of or, rather than two separate accounts.
- This is difficult if statements and questions have different types of meaning.

2. Uniform account of logical operations

- Many operators can embed both statements and questions.

(9) a. Alice rented a car and Bob booked a hotel.
b. Where can we rent a car and which hotel should we get?

(10) a. Alice will rent a car, or she will borrow Bob's car.
b. Where can we rent a car, or whose car can we borrow?

(11) a. If Bob goes to London, he'll stay with Alice.
b. If Bob goes to London, where will he stay?

(12) a. Bob knows that Alice lives in London.
b. Bob knows where Alice lives.

- Standard analyses of and, or, if, know only capture their role in the a-sentences.
- More general accounts of these items come within reach if we analyze statements and questions with a single notion of meaning.

3. Refining the semantics of statements

- To account for the role of or in disjunctive questions we need a more fine-grained representation of disjunctions than the standard one, one that doesn't conflate the propositions expressed by the disjuncts.
- In fact, it seems that this fine-grained representation of disjunctions is needed independently of questions.
- E.g., (13) does not merely provide a single disjunctive permission, but two permissions, corresponding to the two disjuncts.

(13) You may go to London or to Paris. $\Diamond(l \vee p) \rightsquigarrow \Diamond l, \Diamond p$

- If the propositions expressed by the disjuncts were unioned by or, they could no longer be accessible to the operator $\Diamond$.

For these reasons, we pursue a single notion of meaning which is general enough to interpret both statements and questions in a uniform way.

Part II
A new notion of meaning

From truth to support

- In order to provide a uniform semantics, we will revise the fundamental semantic notion.

$$\begin{array}{rcl} \text{truth} & @ & \text{state of affairs} \\ & \Downarrow & \\ \text{support} & @ & \text{state of information} \end{array}$$

- Notation: $s \models \varphi$
- Idea: s supports φ iff φ is settled given the information in s

Information states

- An information state s is modeled as a set of possible worlds.
- The information in s is compatible with the worlds $w \in s$, and incompatible with the worlds $w \notin s$.
- An info state t contains at least as much information as s if $t \subseteq s$. We say that t is an enhancement of s.
- Limit cases:
  - W, the weakest state, called the ignorant state
  - $\emptyset$, the strongest state, called the inconsistent state
  - $\{w\}$, a maximal consistent state, called a complete state

Support: illustration

Statements:

A_1 Alice lives in Paris

A_2 Alice does not live in Paris

Questions:

Q_1 Where Alice lives

Q_2 whether Alice lives in Paris

Propositions

- In the std approach, the fundamental notion is truth at a world. The proposition expressed by φ is the set of worlds where φ is true:

$$|\varphi| = \{w \in W \mid \varphi \text{ is true at } w\}$$

- In InqSem, the fundamental notion is support at an info state. The proposition expressed by φ is the set of states supporting φ:

$$[\varphi] = \{s \subseteq W \mid \varphi \text{ is supported at } s\}$$

- The maximal elements in $[\varphi]$ are called the alternatives for φ.

Propositions

- We assume that support satisfies the following general properties:
  - Persistency: if $s \models \varphi$ and $t \subseteq s$ then $t \models \psi$
  - Supportability: $s \models \varphi$ for some information state s
- As a consequence, $[\varphi]$ is always a non-empty set satisfying:
  - Downward closure: if $t \subseteq s \in [\varphi]$ then $t \in [\varphi]$
- Abstracting away from sentences, we get at the kind of structure which is the fundamental carrier of meaning in inquisitive semantics.

Definition

A proposition is a non-empty set P of information states which satisfies:

- Downward closure: if $t \subseteq s \in P$ then $t \in P$

The set of all propositions is denoted $\mathcal{P}$.

- Standardly, a proposition captures a certain informative content.
- In InqSem, a proposition P captures two aspects of meaning:
  - informative content: $\text{info}(P) = \bigcup P$
    P encodes the information that the world is located in $\text{info}(P)$
  - inquisitive content: a specific set of enhancements of $\text{info}(P)$
    P encodes an issue which is resolved only in the states $s \in P$
- Either component may be trivial:
  - P is non-informative if $\text{info}(P) = W$
  - P is non-inquisitive if $\text{info}(P) \in P$ (equivalently: if $P = \{\text{info}(P)\}^{\downarrow}$)
- In the context of a finite universe of possible worlds:
  - P is non-inquisitive if it has a unique alternative;
  - P is inquisitive if it has two or more alternatives.

Entailment

Entailment in InqSem is defined as preservation of support:

$$\varphi \models \psi \iff \forall s \subseteq W: \; s \models \varphi \text{ implies } s \models \psi$$

Since support is defined for both statements and questions, both can now meaningfully take part in entailment relations.

Statement-to-statement

Coincides with standard truth-conditional entailment.

Statement-to-question

$S \models Q \;\approx\; S$ resolves Q

A doesn't live in Paris $\models$ whether A lives in Paris
$\not\models$ where A lives

Question-to-statement

$Q \models S \;\approx\; Q$ presupposes S

where in Europe A lives $\models$ A lives in Europe

Question-to-question

$Q \models Q' \;\approx\; Q$ determines Q'

where A lives $\models$ whether A lives in Paris
whether A lives in Paris $\not\models$ where A lives

- The inquisitive perspective allows us to generalize the notion of entailment to encompass questions.
- Once we do so, various interesting logical notions turn out to be facets of entailment.
- This allows us to treat these notions using the tools of logic.
- For instance we can now give logical proofs that show that a question determines another.

Notice that:

$$\begin{aligned} \varphi \models \psi \quad & \iff \quad \forall s: \; s \models \varphi \text{ implies } s \models \psi \\ & \iff \quad \forall s: \; s \in [\varphi] \text{ implies } s \in [\psi] \\ & \iff \quad [\varphi] \subseteq [\psi] \end{aligned}$$

Thus, as in the classical case, entailment is inclusion of propositions.
We can also apply the notion of entailment to propositions themselves.

Entailment on propositions
Let P, Q be two InqSem propositions:

$$P \models Q \overset{def}{\iff} P \subseteq Q$$

Entailment partially orders propositions in terms of strength.
This determines the natural treatment of logical operations in InqSem.

Part III

Logical operations in inquisitive semantics

The algebraic approach

- Both truth-conditional semantics and inquisitive semantics provide:
  - a set $\mathcal{P}$ of propositions, to serve as contents of sentences;
  - a partial order $\models$ on $\mathcal{P}$, which allows us to compare meanings.
- Let us call a pair $\langle \mathcal{P}, \models \rangle$ a semantic space.
- A principled way of defining logical operators:
  - look at the algebraic operations which are defined on $\langle \mathcal{P}, \models \rangle$;
  - take logical constants to perform these operations.
- Let us illustrate this by looking at classical logic.

Classical semantic space

The classical semantic space is $\mathcal{S}_{\mathsf{cl}} = \langle \mathcal{P}_{\mathsf{cl}}, \models_{\mathsf{cl}} \rangle$ where:

- $\mathcal{P}_{\mathsf{cl}} = \wp(W)$
- $\models_{\mathsf{cl}} = \subseteq$

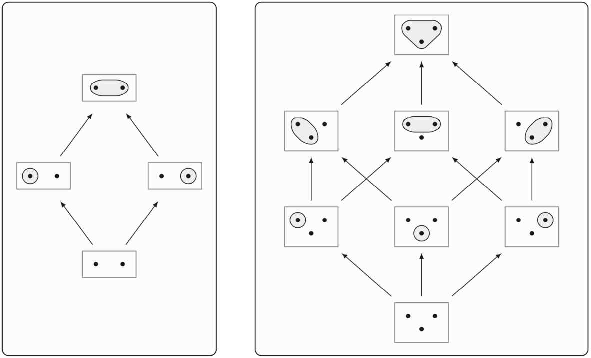

FIGURE 3.1 The set of all propositions in classical semantics if the logical space consists of two possible worlds (on the left) or three possible worlds (on the right). Arrows indicate entailment.

$\mathcal{S}_{\text{cl}}$ is a bounded lattice

- There are a least proposition, $\emptyset$, and a greatest proposition, W.
- $\forall p, q$ we have a meet, i.e., a greatest r such that $r \models p$ and $r \models q$: this is $p \cap q$
- $\forall p, q$ we have a join, i.e., a least r such that $p \models r$ and $q \models r$: this is $p \cup q$

$\mathcal{S}_{\text{cl}}$ is a Heyting algebra

- $\forall p, q$ we have an implication of $p \Rightarrow q$, a greatest r s.t. $p \cap r \models q$: this is $p \Rightarrow q := \overline{p} \cup q$
- $\forall p$ we have a pseudo-complement p^*, a greatest r s.t. $p \cap r \models \emptyset$: this is $p^* = \overline{p}$

$\mathcal{S}_{\text{cl}}$ is a Boolean algebra

- $\forall p$, p^* is a Boolean complement: $p \cup p^* = W$

Classical logic associates the connectives with the algebraic operations:

- $|\varphi \wedge \psi| = |\varphi| \cap |\psi|$
- $|\varphi \vee \psi| = |\varphi| \cup |\psi|$
- $|\varphi \rightarrow \psi| = |\varphi| \Rightarrow |\psi|$
- $|\neg\varphi| = |\varphi|^*$

Inquisitive semantic space

Let us now look at the structure of the space $\mathcal{S}_{\mathsf{inq}} = \langle \mathcal{P}_{\mathsf{inq}}, \models_{\mathsf{inq}} \rangle$, where:

- $\mathcal{P}_{\mathsf{inq}}$ is the set of inquisitive propositions
- $\models_{\mathsf{inq}} = \subseteq$

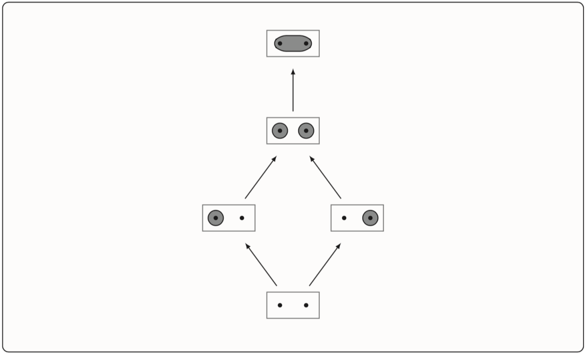

FIGURE 3.3 The set of all inquisitive semantics propositions if the logical space consists of two possible worlds. Arrows indicate entailment.

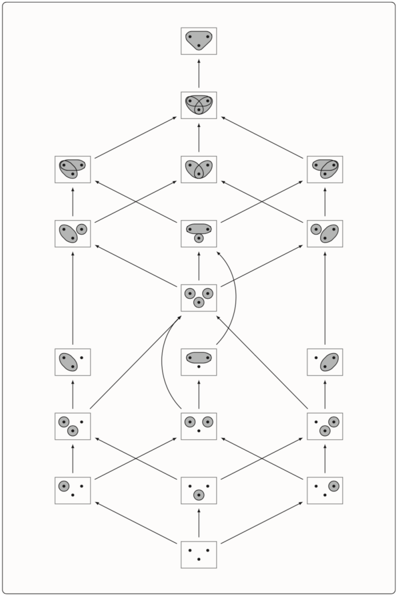

Figure 3.4 The set of all inquisitive semantics propositions if the logical space consists of three possible worlds. Arrows indicate entailment.

$\mathcal{S}_{\text{inq}}$ is a bounded lattice

- The space has a bottom element, $\{\emptyset\}$, and a top element, $\wp(W)$.
- Every $P, Q \in \mathcal{P}_{\text{inq}}$ have a meet: $P \cap Q$
- Every $P, Q \in \mathcal{P}_{\text{inq}}$ have a join: $P \cup Q$

$\mathcal{S}_{\text{inq}}$ is a Heyting algebra

- $\forall P, Q \in \mathcal{P}_{\text{inq}}$ there is an implication of P and Q:

$$P \Rightarrow Q \;=\; \{s \mid \forall t \subseteq s : t \in P \text{ implies } t \in Q\}$$

- $\forall P \in \mathcal{P}_{\text{inq}}$ there is a pseudo-complement of P:

$$P^* \quad = \quad P \Rightarrow \{\emptyset\} \;=\; \{\overline{\text{info}(P)}\}^{\downarrow}$$

$\mathcal{S}_{\text{inq}}$ is not a Boolean algebra

- In general, $P \cup P^*$ is not the top element of the algebra.

Connectives in inquisitive logic

Since we still have meets, joins, implications, pseudo-complements, we can preserve the essence of the classical treatment of connectives:

- $[\varphi \wedge \psi] = [\varphi] \cap [\psi]$
- $[\varphi \vee \psi] = [\varphi] \cup [\psi]$
- $[\varphi \rightarrow \psi] = [\varphi] \Rightarrow [\psi]$
- $[\neg\varphi] = [\varphi]^*$

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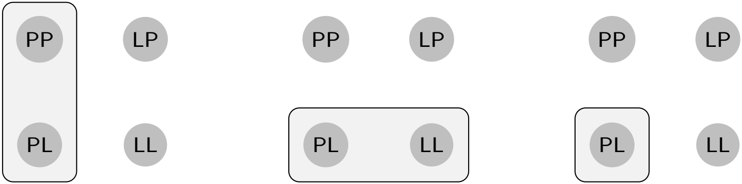
- $[\varphi \wedge \psi] = [\varphi] \cap [\psi]$
- $[\varphi \vee \psi] = [\varphi] \cup [\psi]$
- $[\varphi \to \psi] = [\varphi] \Rightarrow [\psi]$
- $[\neg\varphi] = [\varphi]^*$

Now we have the inquisitive counterpart of classical logic.
Does this relate to how “and”, “or” etc. work in natural language?

Conjunction

$[\varphi \wedge \psi] = [\varphi] \cap [\psi]$

(14) Alice lives in Paris and Bob lives in London.



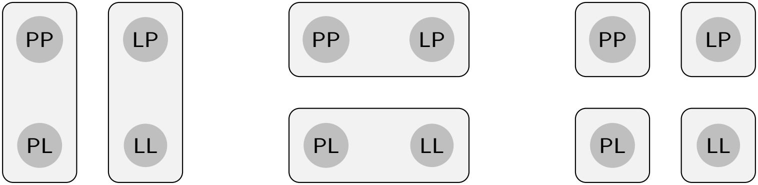
(a) A lives in Paris (b) B lives in London (c) conjunction

In general, on statements, conjunction works in the standard way.

Conjunction

$[\varphi \wedge \psi] = [\varphi] \cap [\psi]$

(15) Where does Alice live, and where does Bob live?



(d) where A lives (e) where B lives (f) conjunction

In general, a conjunctive question is settled iff both conjuncts are.

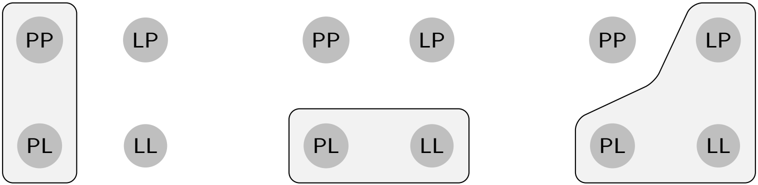
So, from the algebraic picture we get an account of conjunction which:

- coincides with the standard one when applied to statement;
- also allows us to interpret the role of conjunction in questions.

Implication

$[\varphi \to \psi] = [\varphi] \Rightarrow [\psi]$

(16) If Alice lives in Paris then Bob lives in London.



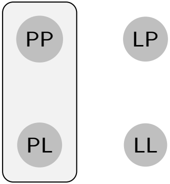
(g) A lives in Paris (h) B lives in London (i) implication

In general, on statements implication behaves in the standard way.

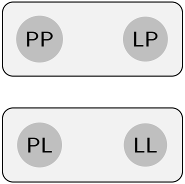
Implication

$[\varphi \wedge \psi] = [\varphi] \Rightarrow [\psi]$

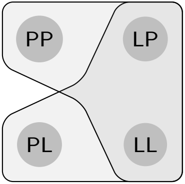
(17) If Alice lives in Paris, where does Bob live?



(j) A lives in Paris



(k) where Bob lives



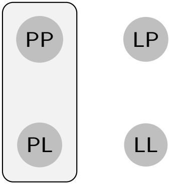
(l) implication

In general, a conditional question if A then Q is settled in a state s if and only if Q is settled relative to the hypothetical state $s \cap |A|$.

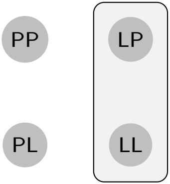
Negation

$[\neg\varphi] = [\varphi]^* = \{\overline{\mathsf{info}(\varphi)}\}^{\downarrow}$

(18) Alice does not live in Paris.



(m) A lives in Paris



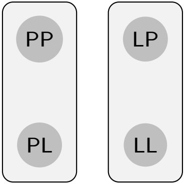
(n) negation

In general, on statements negation behaves in the standard way.

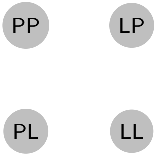
Negation

$[\neg\varphi] = [\varphi]^* = \{\overline{\text{info}(\varphi)}\}^{\downarrow}$

(19) *It is not the case where Alice lives.



(o) where A lives

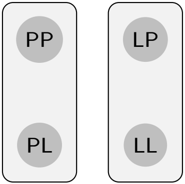


(p) negation (contradictory)

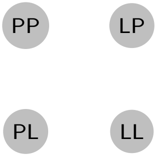
Negation

$[\neg\varphi] = [\varphi]^* = \{\overline{\mathsf{info}(\varphi)}\}^{\downarrow}$

(19) *It is not the case where Alice lives.



(q) where A lives



(r) negation (contradictory)

The fact that negating questions systematically produces a contradiction could be why natural languages do not allow negations of questions.

Similar accounts of the ungrammaticality of certain patterns in terms of systematic semantic triviality have been offered for various puzzles.
(see: Barwise&Cooper 81, von Fintel 93, Fox&Hackl 07, Gajewski 08, Chierchia 13)

(20)
a. There are some cookies in the jar.
b. There is no cookie in the jar.
c. *There are most cookies in the jar.
d. *There is every cookie in the jar.

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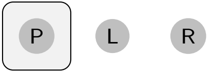
(20)
a. There are some cookies in the jar.
b. There is no cookie in the jar.
c. *There are most cookies in the jar.
d. *There is every cookie in the jar.

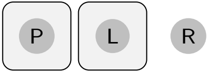
Warning!
I am cheating a bit here: in order to predict that negating a question *always* yields a contradiction, we must take into account presuppositions.

Disjunction

$[\varphi \vee \psi] = [\varphi] \cup [\psi]$

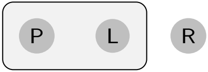


(s) A lives in Paris



(t) inquisitive disjunction

(u) A lives in London



(v) classical disjunction

Even on non-inquisitive propositions, inquisitive $\vee$ behaves differently from classical $\vee$. It yields a proposition with two alternatives.

Alternative-generating disjunction 1

Disjunction is involved in the formation of both statements and questions.

(21) Alice lives in Paris, or she lives in London.

(22) Does Alice live in Paris, or does she live in London?

(w) Statement (x) Question

- As we will see today, in InqSem the contribution of disjunction in these sentences can be analyzed uniformly as $\vee$.
- The differences can be attributed to the semantic contribution of clause-type marking (declarative vs. interrogative).

Alternative-generating disjunction 2

- Various authors (Simons 05, Alonso-Ovalle06, Aloni 07) have argued independently that disjunction should not be represented as an ordinary connective, taking two propositions to a proposition.
- Rather, they take disjunction to deliver a set containing the propositions expressed by the disjuncts: $[\varphi \vee \psi] = \{|\varphi|, |\psi|\}$.
- In this way, the propositions expressed by the disjuncts remain visible to further operators, which can manipulate each of them separately.
- By contrast, if disjunction unions two propositions, a further operator can no longer see the parts out of which the union was formed.

Alternative-generating disjunction 2

- This treatment was motivated empirically: it was used, e.g., to account for the interpretation of "or" in modals and conditionals.
- This is a sharp departure from the treatment of disjunction as join.
- It particular, it is not clear how the resulting sets of propositions can be compared with ordinary propositions in terms of entailment, or how they should be handled by other connectives like $\neg$.

Reconciliation

In InqSem, the two perspectives on disjunction can be reconciled.

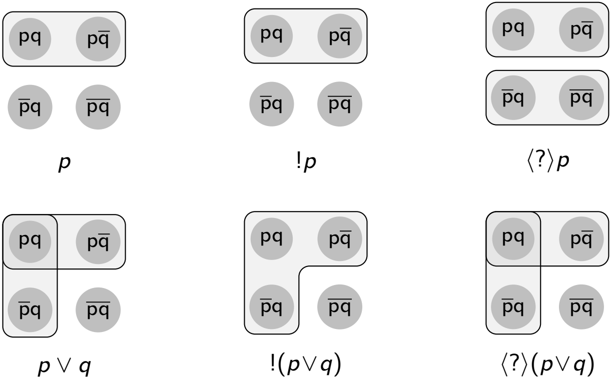
- Disjunction does not require an exceptional treatment: all connectives are operations on inquisitive propositions.
- Like in classical logic, disjunction is treated as a join operator. This guarantees that we preserve its fundamental logical properties.
- At the same time, the meaning of the individual disjuncts typically remains visible in the proposition expressed by the disjunction, and it is accessible to further operators.
- Thus, we don't have to choose between the algebraic and the alternative-generating treatment of $\vee$: the two go hand-in-hand.
- Moreover, the fact that $\vee$ generates alternatives follows from considerations independent of the linguistic data to be explained. This makes the account less stipulative and more explanatory.

Non-inquisitive projection

- $[!\varphi] = [\varphi]^{**} = \{\text{info}(\varphi)\}^{\downarrow}$

Inquisitive projection

- $[\langle ? \rangle \varphi] := \begin{cases} [\varphi] & \text{if } [\varphi] \text{ is inquisitive} \\ [\varphi] \cup [\varphi]^{*} & \text{otherwise} \end{cases}$



Part IV

Embedding questions: an illustration

Many verbs, including know, remember, guess, tell, discover, ...
can embed both declarative and interrogative complements.

(23)
a. Bob knows that Alice is French.
b. Bob knows that Alice is French or German.
c. Bob knows whether Alice is French.
d. Bob knows whether Alice is French or German.

Know: the standard analysis

- Bob's epistemic state at w, $\sigma_b(w)$, is the set of worlds compatible with what Bob knows at w.
- The knowledge operator has the following truth-conditions:

$$w \models K_b(\varphi) \iff \sigma_b(w) \subseteq |\varphi|$$

- This analysis assumes that the argument φ is truth-conditional. Thus, it allows us to interpret (24-a,b) but not (24-c,d).

(24)
  a. Bob knows that Alice is French.
  b. Bob knows that Alice is French or German.
  c. Bob knows whether Alice is French.
  d. Bob knows whether Alice is French or German.

Know: the inquisitive analysis

- We can generalize the truth-conditions for the knowledge operator:

$$w \models K_b\varphi \iff \sigma_b(w) \models \varphi$$

- The inquisitive proposition associated with a knowledge statement is non-inquisitive:

$$[K_b\varphi] = \{|K_b\varphi|\}^{\downarrow}$$

If we add K to our logical language and plug in our translations of the embedded sentences we get:

(25)	Bob knows that Alice is French.	$K_b!\widehat{Fa}$
(26)	Bob knows that Alice is French or German.	$K_b!(\widehat{Fa} \vee \widehat{Ga})$
(27)	Bob knows whether Alice is French.	$K_b\langle ? \rangle\widehat{Fa}$
(28)	Bob knows whether Alice is French or German.	$K_b\langle ? \rangle(\widehat{Fa} \vee \widehat{Ga})$

(29) Bob knows that Alice is French. $\rightsquigarrow$ $K_b!\widehat{Fa}$

$$w \models K_b!\widehat{Fa} \iff \sigma_b(w) \models !\widehat{Fa} \iff \sigma_b(w) \subseteq |Fa|$$

(30) Bob knows that Alice is French or German. $\rightsquigarrow$ $K_b!(\widehat{Fa} \vee \widehat{Ga})$

$$w \models K_b!(\widehat{Fa} \vee \widehat{Ga}) \iff \sigma_b(w) \models !(\widehat{Fa} \vee \widehat{Ga}) \iff \sigma_b(w) \subseteq |Fa| \cup |Ga|$$

(31) Bob knows whether Alice is French. $\rightsquigarrow K_b\langle ? \rangle \widehat{Fa}$

$$w \models K_b\langle ? \rangle \widehat{Fa} \iff \sigma_b(w) \models \langle ? \rangle \widehat{Fa}$$
$$\iff \sigma_b(w) \subseteq |Fa| \text{ or } \sigma_b(w) \subseteq \overline{|Fa|}$$

(32) Bob knows whether A is French or German. $\rightsquigarrow K_b\langle ? \rangle (\widehat{Fa} \vee \widehat{Ga})$

$$w \models K_b\langle ? \rangle (\widehat{Fa} \vee \widehat{Ga}) \iff \sigma_b(w) \models \langle ? \rangle (\widehat{Fa} \vee \widehat{Ga})$$
$$\iff \sigma_b(w) \subseteq |Fa| \text{ or } \sigma_b(w) \subseteq |Ga|$$

(31) Bob knows whether Alice is French. $\rightsquigarrow$ $K_b\langle ? \rangle \widehat{Fa}$

$$w \models K_b\langle ? \rangle \widehat{Fa} \iff \sigma_b(w) \models \langle ? \rangle \widehat{Fa}$$
$$\iff \sigma_b(w) \subseteq |Fa| \text{ or } \sigma_b(w) \subseteq \overline{|Fa|}$$

(32) Bob knows whether A is French or German. $\rightsquigarrow$ $K_b\langle ? \rangle (\widehat{Fa} \vee \widehat{Ga})$

$$w \models K_b\langle ? \rangle (\widehat{Fa} \vee \widehat{Ga}) \iff \sigma_b(w) \models \langle ? \rangle (\widehat{Fa} \vee \widehat{Ga})$$
$$\iff \sigma_b(w) \subseteq |Fa| \text{ or } \sigma_b(w) \subseteq |Ga|$$

So, InqSem allows for an elegant extension of the std account of know which applies uniformly to statements and question.