Introduction to natural language semantics
Class 10: indexicals

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Indexicals: basic observations

- In intensional semantics we have taken sentences to express propositions, i.e., functions p from possible worlds to truth-values.
- Such a function can be identified with a set of worlds, $s_p = \{w \in W \mid p(w) = 1\}$.
- For instance (1-a) expresses the proposition (1-b).

(1) a. Alice is Irish.
    b. $p = \{w \in W \mid \text{Alice is Irish in } w\}$

- A complication arises when we consider *indexical* expressions, including:
  - individual: I, we, you (singular and plural)
  - temporal: now, later, earlier, today, yesterday, tomorrow
  - spatial: here, there
  - modal: actually
- To see the problem, let us ask: what is the proposition expressed by (2)?

(2) I am Irish.

- It depends on who is uttering the sentence. If the speaker is Alice, then she could have expressed the same content by uttering (1); but if the speaker is Bob, he could have expressed the same proposition by uttering (3).

(3) Bob is Irish.

- In other words: when (3) is uttered by Alice it expresses p; when it is uttered by Bob is expresses q.

(4) a. $p = \{w \in W \mid \text{Alice is Irish in } w\}$
    b. $q = \{w \in W \mid \text{Bob is Irish in } w\}$

- So, there is no single proposition expressed by (4). Rather, (4) expresses different propositions in different *contexts of utterance*.
- How to handle this complication?

Two-dimensional semantics

- Add the context of utterance as a new parameter to the semantics
- A context of utterance c consists of various things:
  - a world w_c where the utterance takes place
  - an agent s_c, the speaker/writer of the utterance
  - optionally, an agent a_c, the adressee
  - a time t_c and place p_c where the utterance takes place
  - etc. (which features we take to be constitutive of a context will depend on which indexicals we need to account for)
- So (2) will have a truth-value relative to two parameters (besides the assignment function, which we are temporarily setting aside since it doesn't play a role here):
  - a context of utterance c, which determines the proposition $[\![\varphi]\!]^c$ expressed;
  - a world of evaluation w at which this proposition may be true or false:

$$[\![\varphi]\!]^{c,w} = [\![\varphi]\!]^c(w)$$

- This kind of architecture is known as *two-dimensional* semantics (2D-semantics).
- In 2D-semantics, there are three levels to the semantics of a sentence φ:
  - its *extension* $[\![\varphi]\!]^{c,w}$ relative to a context c and a world of evaluation w: a truth-value;
  - its *content/intension* $[\![\varphi]\!]^c$ relative to a context c: a function from worlds to truth-values, i.e., a proposition;
  - its *character* $[\![\varphi]\!]$: a function from contexts to propositions.
- We have different notions of equivalence. Imagine Alice says:

  (5) I am Irish.

  Now imagine Bob says to Alice:

  (6) a. You are Irish.
  b. I am Irish.

  In which case is Bob saying the same thing as Alice?
- It depends on what we mean by "saying the same":

- Content equivalence in the respective contexts:
  when Bob utters (6-a), he expresses the same content as Alice does with (5) (although he must use an expression with different character to do so)
- Character equivalence:
  when Bob utters (6-a), he expresses a different content than Alice expressed, but he does so by using the same character that Alice used.

- The three-level distinction between *character*, *content/intension*, and *extension* must be made not just for sentences, but for expressions of all semantic categories.
- E.g., suppose α is an individual-denoting expression (type e).
  Then we have:
  - its *extension* $[\![\varphi]\!]^{c,w}$ relative to a context c and a world w: an individual;
  - its *content/intension* $[\![\varphi]\!]^{c}$ relative to a context of utterance c:
    a function from worlds to individuals, i.e., an individual concept;
  - its *character* $[\![\varphi]\!]$: a function from contexts to individual concepts.
- Illustration (on the board):

(7) $[\![\text{the US president}]\!]^{c}$ = the function mapping w to the US president at w

(8) $[\![\text{I}]\!]^{c}$ = the function mapping w to the speaker of c

- The former has constant character; the latter has constant content in all contexts.
- We can put the situation like this:
  - an indexical like 'I' gets its referent from the context of utterance (row);
  - a non-indexical like 'the US President' gets its referent from the world of evaluation (column).

Truth simpliciter

- Now suppose Donald Trump says:

(9) I am the US President.

- Intuitively, this utterance is *true*. What does this mean?
- It means that it is true at the specific world where the utterance took place, w_c
- When we ask an utterance of a sentence in a context c is true or false *simpliciter*, we mean whether it's true or false in the world w_c where the utterance took place.
- Thus, an important role is played by the *diagonal* truth-value at c:

$$[\![\varphi]\!]^{c,w_c}$$

- Thus, e.g.,

$$[\![\text{I am the US President}]\!]^{c,w_c} = \begin{cases} 1 & \text{if } s_c \text{ is the US President in } w_c \\ 0 & \text{if } s_c \text{ is not the US President in } w_c \end{cases}$$

Logical truth vs. necessary truth

- Adding indexicals to the picture gives rise to surprising novelties.
- Consider (10):

  (10) I am here.

- There is a sense in which this is a fact of the logic of indexicals: if we understand what 'I' and 'here' mean, we know a priori that (10) must be true in every context of utterance, regardless of the particular circumstances.
- Yet, (10) does *not* express a necessary proposition. When I say 'I am here', what I am saying is something that could have been false; I could have been elsewhere.
- For instance, in some context of utterance, it expresses the same proposition as the following sentence, which is certainly not a logical validity.

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- In sum, (10) is logically valid, yet it expresses a contingent proposition.
- How can this be?
- Another way to put this: (12) is false.

  (12) It is necessary that I be here.

- This goes against one feature of standard modal logic:
  - Necessitation: if φ is logically valid, then $\Box\varphi$ is logically valid.
- How to make sense of these observations?
- Following Kaplan, let us define validity as diagonal truth at every context.
  - Validity: $\models \varphi \iff \forall c : [\![\varphi]\!]^{c,w_c} = 1$
- Notice that validity just looks at pairs $\langle c, w_c \rangle$, not at all pairs $\langle c, w \rangle$ allowed by the semantics.
- Thus, even though φ is valid, it could be that $[\![\varphi]\!]^{c,w} = 0$ for some c, w.
- If so, the proposition expressed by φ in context c, although true at the actual world w_c of the context, is nevertheless contingent, since it is false at other worlds.

Formalization

- Let's see how we can make this formally precise in a logical system.
- I will focus on adding the tools to model the indexicals 'I', 'here', and 'actually', but the strategy is general.
- Language: the one of intensional type theory as we saw it in Class 8, plus some new constants:
  - $i : e$ for 'I'.
  - $h : e$ for 'here'.
  - $L : \langle e, \langle e, t\rangle\rangle$ for 'located'.

  and some new sentential operators:

  - N for 'necessarily'
  - A for 'actually'
- A model is a tuple $M = \langle D, W, C, I\rangle$ where:
  - D, W are non-empty sets (of individuals and worlds, respectively)
  - C is a set of contexts, where a context is a triple $c = \langle w_c, s_c, p_c\rangle$ where:
    * $w_c \in W$ is the world of c
    * $s_c \in D$ is the speaker of c
    * $p_c \in D$ is the place of c
  - I is a function that to every context c, world w, and constant $a \in \mathrm{Con}(\tau)$ assigns a denotation/extension $I_{c,w}(a) \in D^M_\tau$, with the following constraints:
    * the denotation of a non-indexicals does not depend on the context: if $a \neq i, h$, then $\forall c, c', w'$: $I_{c,w}(a) = I_{c',w}(a)$
    * 'I' denotes the speaker of the context: $I_{c,w}(i) = s_c$
    * 'here' denotes the place of the context: $I_{c,w}(h) = p_c$
    * in the world w_c, s_c is located at p_c: for every c: $\langle s_c, p_c\rangle \in I_{c,w_c}(L)$
- The interpretation function $[\![\cdot]\!]^{M,g,c,w}$ is defined as:
  - if a is a constant then $[\![a]\!]^{M,g,c,w} = I_{c,w}(a)$
  - if x is a variable then $[\![x]\!]^{M,g,c,w} = g(x)$
  - other clauses remain essentially as in standard intensional semantics
  - $[\![N\varphi]\!]^{M,g,c,w} = 1 \iff [\![\varphi]\!]^{M,g,c,w'} = 1$ for all $w' \in W$
  - $[\![A\varphi]\!]^{M,g,c,w} = 1 \iff [\![\varphi]\!]^{M,g,c,w_c} = 1$

- Logical validity, diagonal definition:

$$\begin{aligned} \models \varphi \quad &\Longleftrightarrow \quad \forall M, g, c : [\![\varphi]\!]^{M,g,c,w_c} = 1 \\ &\not\Longleftrightarrow \quad \forall M, g, c, w : [\![\varphi]\!]^{M,g,c,w} = 1 \end{aligned}$$

- Now we can check that
  - $L(i, h)$ ("I am here") is logically valid
  - $N(L(i, h))$ ("it is necessary that I be here") is not logically valid
- Another interesting feature:
  - φ and $A\varphi$ are logically equivalent:
    - $\models \varphi \leftrightarrow A\varphi$
    - since for every M, g, c: $[\![A\varphi]\!]^{M,g,c,w_c} = [\![\varphi]\!]^{M,g,c,w_c}$
  - but φ and $A\varphi$ are not semantically equivalent:
    - since for some M, g, c, w: $[\![A\varphi]\!]^{M,g,c,w} \neq [\![\varphi]\!]^{M,g,c,w}$
  - and they are not inter-substitutable:
    - $\not\models NA\varphi \leftrightarrow N\varphi$
  - thus, the resulting logic does not allow replacement of logical equivalents (though it does allow replacement of *semantic* equivalents).
- By using the indexical operator A we can render some natural language sentences that cannot be expressed in standard modal logic.
- Consider (13):

(13) Everyone who is actually inside might be outside.

- We are interested in the reading where the modal takes wide scope, which can be expressed semi-formally as:

(14) There is a possible world v such that everyone who is inside in the actual world is outside in v

- This was not expressible in standard intensional semantics, regardless of how the modal *might* and the quantifier *everyone* take relative scope:
  - $\Diamond\forall x(Ix \to Ox)$
  - $\forall x(Ix \to \Diamond Ox)$
- Now we have the means to express it:
  - $\Diamond\forall x(A(Ix) \to Ox)$
  - $[\![\Diamond\forall x(A(Ix) \to Ox)]\!]^{M,c,w_c} = 1 \iff \exists v \in W \forall d \in D : [\![Id]\!]^{M,c,w_c}$ implies $[\![Od]\!]^{M,c,v}$
- In standard modal logic, once we leave a world of evaluation with a modal, no operator can take us back to it to evaluate a sub-formula.
- This is precisely what the indexical operator A allows us to do.