Minimal-change counterfactuals in intuitionistic logic

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Abstract. In this paper we study the logic IVC obtained by adding Lewis-style counterfactual conditionals to intuitionistic propositional logic. Building on recent work by Weiss [21], we first show how to introduce a Lewisian counterfactual operator into intuitionistic Kripke semantics. We then establish a complete axiomatization of the resulting logic.

Keywords: counterfactuals · intuitionistic logic · variably strict conditionals · minimal-change semantics · non-monotonic reasoning

1 Introduction

David Lewis [14] proposed an analysis of counterfactuals within modal logic which remains the most influential logical approach to counterfactuals to date. In this semantics, a counterfactual $\varphi > \psi$ is seen as a kind of box modality, which asserts that the consequent holds throughout a certain set $f(w, \varphi)$ of possible worlds, depending on the world of evaluation and, crucially, on the antecedent. Lewis' intuition is that the worlds in $f(w, \varphi)$ are those worlds in which φ is true, and which are otherwise minimally different from w.[3] This intuitive characterization yields certain constraints on the behavior of the selection function f, which give rise to a specific logic of counterfactuals, called VC. This logic, though not without its issues,[4] does a remarkable job at accounting for the ways in which the logical behavior of counterfactuals differs from that of the implication connective in classical (and intuitionistic) logic: for counterfactuals, VC invalidates strengthening of the antecedent, transitivity, and contraposition.

[3] This description of Lewis's view presupposes the *limit assumption*, i.e., the assumption that for any w and any entertainable φ there be worlds where φ is true and which differ minimally from w in the relevant sense. In our study we will take this assumption for granted, for two reasons. First, this assumption allows for a nice characterization of the semantics in terms of selection functions, and does not affect the resulting propositional logic. Second, there are in fact good conceptual reasons to make the limit assumption: as [12] showed, this assumption is needed to guarantee that an entertainable antecedent has a consistent set of counterfactual consequences.

[4] For criticism of different aspects of this logic, see, e.g., [9, 2, 6, 8, 16].

And for a good reason, since both Lewis [14] and Stalnaker [19] give examples where these principles seem to be fallacious.

In the decades following Lewis' work, VC and its relatives have been thoroughly investigated from the perspective of modal logic (see [15] for an overview), and connected to a number of other topics in logic such as probabilistic reasoning [1], belief revision [11, 5, 3] and default reasoning [13, 20]. This body of work focuses on adding counterfactual-like operators against the background of classical propositional logic. However, there seems to be no special reason why the study of such operators should be restricted to a classical setting. The central ideas of Lewis's account are fully modular with respect to the specific semantics used to interpret the constituents of a counterfactual, as long as an intensional setting is available. Thus, it seems interesting to look at how Lewisian counterfactuals could be added to various non-classical logics. This is particularly natural in the case of intuitionistic propositional logic, since the most commonly used semantics for this logic, intuitionistic Kripke semantics, is already intensional in nature, and therefore provides an ideal environment to implement Lewis' idea.

Somewhat surprisingly, a study of Lewisian counterfactuals in intuitionistic Kripke semantics is missing in the literature. Recently, Weiss [21] took a first step in this direction, showing how to extend intuitionistic Kripke semantics with the structure needed to interpret a non-monotonic conditional operator $>$, and studying the intuitionistic counterparts of some very weak conditional logics, including the minimal logic CK of Chellas [7]. These weak logics are interesting, as they bring out most clearly the connection between counterfactuals and modal operators. However, they are generally regarded as too weak to capture many interesting logical principles about counterfactuals. For instance, they *never* allow strengthening of the antecedent, which seems too restrictive. In this paper, we follow up on Weiss' work by studying the intuitionistic counterpart of Lewis's logic VC. In order to achieve this goal, we will propose a semantics that departs slightly from the one given by Weiss. The modifications do not affect the generality of the semantics: it can be shown that a Weiss model can be turned into one of our models without affecting the satisfaction of formulas, and vice versa. However, the modified notion of models will facilitate a simple and elegant correspondence between semantic conditions and syntactic axioms, which seems hard to obtain using Weiss' original semantics.

Before delving into the technical material, let us mention two reasons why adding counterfactuals in an intuitionistic setting is an interesting enterprise. First, intuitionistic logic is already equipped with its own conditional operator $\to$. Unlike the material conditional of classical logic, which is truth-functional, the conditional of intuitionistic logic is an intensional operator, and its semantics is very similar to the one we will use for the operator $>$: both check whether the consequent is satisfied everywhere within a certain set of possible worlds determined by the antecedent. The difference is which set of worlds is picked out by each of them. Crucially, $\to$ is constrained to quantifying over states of affairs which are *possible* from the perspective of the evaluation world, while $>$ is allowed to quantify over counterfactual states of affairs as well. A natural interpretation is to view $p \to q$ as standing for an indicative conditional like (1-a), and $p > q$ as standing for a counterfactual conditional like (1-b).

(1) a. If the butler didn't do it, the gardener did.
b. If the butler had not done it, the gardener would have.

Second, studying the principles of conditional logic from the perspective of intuitionistic logic allows us to ask which of these principles stem only from assumptions about the semantics of counterfactuals, and which stem partly from the classicality of the logic. As an example, consider a principle central to Lewis' logic, the *rational monotonicity* principle. Informally, this principle says that, if in making a counterfactual assumption φ one leaves open the possibility that ψ, then one is justified in strengthening the antecedent from φ to $\varphi \wedge \psi$. Classically, this principle could be equivalently formulated in either of the following ways:

- $((\varphi > \chi) \wedge \neg(\varphi > \neg\psi)) \to (\varphi \wedge \psi > \chi)$
- $(\varphi > \chi) \to ((\varphi > \neg\psi) \vee (\varphi \wedge \psi > \chi))$

It turns out that, in the intuitionistic setting, the latter, and not the former, is the appropriate way to capture the rational monotonicity constraint. Thus, the former can be seen as a consequence of rational monotonicity plus classical logic.

The paper is structured as follows: in Section 2 we describe how to extend intuitionistic Kripke semantics with the structure needed to interpret a counterfactual conditional operator; in Section 3 we give intuitionistic versions of the assumptions of minimal change semantics, which lead to IVC, an intuitionistic counterpart of Lewis' logic VC; in Section 4 we describe an axiomatization of this logic and show that it is sound; in Section 5 we describe how to construct a canonical model for IVC, and use this construction to prove completeness; Section 6 summarizes our findings and outlines some directions for further work.

2 Counterfactuals in intuitionistic Kripke semantics

In this section we describe how to extend intuitionistic Kripke semantics with the structure needed to interpret a counterfactual conditional operator $>$. The idea is to enrich a Kripke structure with a selection function, which picks for each world and each antecedent a set of "relevant antecedent worlds".

Definition 1 (Intuitionistic selection models).
An intuitionistic selection model is a tuple $M = \langle W, \leq, \mathcal{A}, f, V\rangle$ where:

- *W is a set, whose elements are called* worlds*.*
- *$\leq$ is a partial order on W, the* refinement *ordering; the set of $\leq$-successors of a world w is denoted $w^{\uparrow}$; in symbols: $w^{\uparrow} := \{v \in W \mid w \leq v\}$.*
- *$Up_{\leq}(W)$ denotes the set of up-sets of W, i.e.:*

$$Up_{\leq}(W) = \{X \subseteq W \mid \forall w, v : w \in X \text{ and } w \leq v \text{ implies } v \in X\}$$

– $\mathcal{A} \subseteq Up_{\leq}(W)$ *is a set of up-sets called* propositions*, which contains* $\emptyset$ *and is closed under union, intersection, and the following operations:*

$$X, Y \mapsto \{w \in W \mid X \cap w^{\uparrow} \subseteq Y\} \qquad X, Y \mapsto \{w \in W \mid f(w, X) \subseteq Y\}$$

– $f : W \times \mathcal{A} \to \wp(W)$*, the* selection function*, is a map assigning to each world* w *and proposition* X *a subset* $f(w, X) \subseteq X$ *(the* relevant X-worlds at w*).*
– $V : \mathcal{P} \to \mathcal{A}$ *is a* valuation function*, assigning to each atom a proposition.*

The refinement ordering and the selection function are required to be linked by the following conditions:

– *Upwards-closure:* $f(w, X) \in Up_{\leq}(W)$ *for any* $w \in W$ *and* $X \in \mathcal{A}$*.*
– *Monotonicity of* f *in the first coordinate: if* $w \leq v$ *then* $f(w, X) \supseteq f(v, X)$*.*

One may think of worlds as partial stages in a process of inquiry. At each world, a sentence φ may or may not have been established. The relation $w \leq v$ means that v is a refinement of w: if this holds, then v establishes everything that w establishes, and possibly more. The closure conditions on $\mathcal{A}$ are needed to ensure that the object $|\varphi|$ expressed by a sentence φ in a model is always a proposition, and thus that the hypothetical context $f(w, |\varphi|)$ needed to interpret counterfactuals with antecedent φ is defined. The intended interpretation of f is that the elements $v \in f(w, X)$ are those worlds which, from the standpoint of w, may have obtained if X had been the case. We refer to $f(w, X)$ as the *hypothetical context* generated at w by the making the counterfactual assumption that X.[5]

The condition that $f(w, X)$ be upwards-closed can be motivated as follows: if $v \in f(w, X)$, then at w we think that, had X been the case, v may have obtained. Since v may evolve into any of its successors, any such point may have obtained if X had been the case. Thus, each successor of v should be in $f(w, X)$.

Finally, the monotonicity condition says that, if $w \leq v$, then the hypothetical context $f(v, X)$ is at least as strong as the context $f(w, X)$. This is a natural constraint: $w \leq v$ means that all the information available at w is also available at v; this includes counterfactual information about how things would be if X were the case; so, any counterfactual possibility u which can be ruled out at w ($u \notin f(w, X)$) can also be ruled out at v ($u \notin f(v, X)$). Thus, $f(v, X) \subseteq f(w, X)$.[6]

[5] In the work of Lewis, the selection function takes formulas, rather than propositions, as its second argument. It would in principle be possible to do the same here. However, the presentation would become more complicated: some conditions in the definition of a model (in particular, the requirement that f should yield the same result when applied to intensionally equivalent formulas) appeal to the semantics of sentences, which in turn is defined with reference to the notion of a model. Letting selection functions take propositions allows us to avoid such seeming circularities.

[6] Our semantics departs from the one recently proposed by Weiss [21] in two ways: first, Weiss does not require $f(w, X)$ to be upwards-closed; second, he requires $f(w, X)$ to be defined for all subsets $X \subseteq W$, not just for a designated set of such subsets. Both differences are important for our completeness result. At the same time, however, a Weiss model can be translated to one of our models, and vice versa, without affecting the satisfaction of formulas. A detailed comparison must be left for another occasion.

The addition of the selection function component to intuitionistic Kripke models allows us to interpret a propositional language extended with a counterfactual conditional connective $>$. More precisely, the language $\mathcal{L}^{>}$ that we will work with is given by the following BNF definition:

$$\varphi ::= p \mid \bot \mid \varphi \wedge \varphi \mid \varphi \vee \varphi \mid \varphi \to \varphi \mid \varphi > \varphi$$

As usual in intuitionistic logic, negation and the biconditional are defined as:

$$\neg\varphi := \varphi \to \bot \qquad \varphi \leftrightarrow \psi := (\varphi \to \psi) \wedge (\psi \to \varphi)$$

Satisfaction relative to a model M and a world w is defined as follows.

Definition 2 (Semantics).

1. $M, w \models p \iff w \in V(p)$
2. $M, w \not\models \bot$
3. $M, w \models \varphi \wedge \psi \iff M, w \models \varphi$ *and* $M, w \models \psi$
4. $M, w \models \varphi \vee \psi \iff M, w \models \varphi$ *or* $M, w \models \psi$
5. $M, w \models \varphi \to \psi \iff \forall v \geq w : M, v \models \varphi$ *implies* $M, v \models \psi$
6. $M, w \models \varphi > \psi \iff \forall v \in f(w, |\varphi|) : M, v \models \psi$

where the set $|\varphi|$*, called the* proposition expressed *by of* φ *in* M*, is defined as:*

– $|\varphi| := \{w \in W \mid M, w \models \varphi\}$

To lighten notation, in the following we will write $f(w, \varphi)$ *instead of* $f(w, |\varphi|)$*.*

Clauses 1-5 are just the standard clauses of intuitionistic Kripke semantics. Clause 6 says that $\varphi > \psi$ is satisfied at w if ψ is satisfied at all the relevant φ-worlds at w; that is, $\varphi > \psi$ is satisfied at w iff ψ holds throughout the hypothetical context generated from making the counterfactual assumption φ at w.

It is interesting to note that the semantics for the two conditionals can be made more parallel than it looks at first. To see this, notice that we can think of those $\leq$-successors of w which satisfy φ as those states which may become actual if φ is established. We can then think that supposing φ as an indicative (as opposed to counterfactual) assumption amounts to imagining that we are in one of these worlds. More technically, let us define a selection function g as $g(w, X) := \{v \in X \mid w \leq v\}$; then the semantics of the intuitionistic conditional $\to$ can also be presented in the selection function format:

– $M, w \models \varphi \to \psi \iff \forall v \in g_{\leq}(w, |\varphi|) : M, v \models \psi$

Thus, the difference between the two conditionals lies not in the mathematical workings of their semantics, but rather in the different selection functions that they invoke, corresponding to the difference between supposing φ an indicative assumption and as a counterfactual assumption.

As usual in intuitionistic Kripke semantics, we have a *persistency* property: whatever is established at a world remains established at any refinement of it.

Proposition 1 (Persistency).
For every ICM M, if $w \leq v$ then $M, w \models \varphi$ implies $M, v \models \varphi$.

Proof. By induction on φ. We only give the inductive step for $\varphi = \psi > \chi$. Suppose $w \leq v$ and $M, w \models \psi > \psi$. This means that $f(w, \psi) \subseteq |\chi|$. By the monotonicity condition we have $f(v, \psi) \subseteq f(w, \psi)$. Therefore also $f(v, \psi) \subseteq |\chi|$, which means that $M, v \models \psi > \chi$. □

It is worth pointing out that classical selection function semantics (e.g., [6, 17]) can be retrieved as a special case: classical selection models can be identified with intuitionistic selection models where the refinement relation $\leq$ is the identity; restricted to these models, our semantic clauses boil down to the classical ones.

3 Minimal change conditions

Lewis's minimal change semantics can be seen as obtained from selection function semantics by imposing some constraints on how the selection function works (see [14]§2.7, [6]§1.2). In this section we propose analogues of these constraints in the intuitionistic setting, and discuss some repercussions of these constraints for the logical behavior of conditionals.

Definition 3 (Intuitionistic minimal change models).
An intuitionistic selection model M is called an intuitionistic minimal change model *if it satisfies the following conditions:*

1. *if $w \in X$ then $w \in f(w, X)$*
2. *if $w \in X$ then $f(w, X) \subseteq w^{\uparrow}$*
3. *if $f(w, X) = \emptyset$ and $Y \subseteq X$ then $f(w, Y) = \emptyset$*
4. *if $Y \subseteq X$ and $f(w, X) \cap Y \neq \emptyset$ then $f(w, Y) \subseteq f(w, X)$*
5. *if $Y \subseteq X$ then $f(w, X) \cap Y \subseteq f(w, Y)$*

Condition 1 is known as the *weak centering* condition: it says that if X is true at w, then w is one of the worlds which might be the case if X were the case. That is, if the antecedent is true, then the consequent must be true in order for the counterfactual to be true. In our intuitionistic setting, this constraint implies another interesting property: at every world w, any refinement of w which satisfies φ is relevant to the truth of a conditional $\varphi > \psi$. To state this precisely, recall that we used the notation $g(w, X)$ for the set of refinements of w which are in X, that is, $g(w, X) = \{v \in X \mid v \geq w\}$. We have the following.

Proposition 2. *If 1 holds in M, then for any w and any X: $g(w, X) \subseteq f(w, X)$.*

Proof. Suppose $v \in g(w, X)$, i.e., $v \geq w$ and $v \in X$. By condition 1, $v \in f(v, X)$. By the monotonicity condition, $f(v, X) \subseteq f(w, X)$. Therefore, $v \in f(w, X)$. □

Since $g(w, \varphi)$ and $f(w, \varphi)$ provide, respectively, the domains of quantifications used to assess $\varphi \rightarrow \psi$ and $\varphi > \psi$, this proposition implies the following corollary.

Proposition 3. *If 1 holds in M, then $M, w \models \varphi > \psi$ implies $M, w \models \varphi \to \psi$.*

Condition 2 says that, if φ is true at w, then no *counterfactual* world—i.e., no world which is not a refinement of w—is relevant to the truth of $\varphi > \psi$ at w. In combination with Condition 1, this gives the *strong centering* condition, which in our setting is formulated as follows.

Proposition 4. *Suppose 1 and 2 hold in M. If $w \in X$, then $f(w, X) = w^{\uparrow}$.*

This condition looks a bit different than the classical strong centering condition, which requires that if $w \in X$ then $f(w, X) = \{w\}$. Note, however, that the classical formulation would not be compatible with the upwards-closure requirement on $f(w, X)$, since $\{w\}$ is not upwards-closed if w is not an endpoint (i.e., if there are proper extensions $v > w$). If $f(w, X)$ includes w, then it must contain the whole set $w^{\uparrow}$; thus, $w^{\uparrow}$ is the smallest hypothetical context which includes w.

Nevertheless, if we look at the special case of classical selection models, that is, models where the relation $\leq$ is the identity, then we retrieve the classical formulation of strong centering: for then $w^{\uparrow} = \{w\}$. Furthermore, in our setting the condition $f(w, \varphi) = w^{\uparrow}$ captures exactly the central idea of strong centering, namely: if φ is true at w, then the only world which is relevant to assessing the truth of a conditional $\varphi > \psi$ is w itself.

Proposition 5. *Suppose conditions 1 and 2 hold in M. If $w \in |\varphi|$, for every ψ we have $M, w \models \varphi > \psi \iff M, w \models \psi$.*

Proof. If 1 and 2 hold and $w \in |\varphi|$, by the previous proposition we have $f(w, \varphi) = w^{\uparrow}$. Suppose $M, w \models \varphi > \psi$. Then $f(w, \varphi) \subseteq |\psi|$, and since $w \in w^{\uparrow} = f(w, \varphi)$ we have $M, w \models \psi$. Conversely, suppose $M, w \models \psi$. By persistency, every $v \geq w$ satisfies ψ, so $w^{\uparrow} \subseteq |\psi|$. Since $f(w, \varphi) = w^{\uparrow}$, it follows that $M, w \models \varphi > \psi$. $\square$

The third condition says that if X cannot be consistently supposed, then any proposition stronger than X cannot be consistently supposed either. This gives:

Proposition 6. *If 3 holds in M, $M, w \models \varphi > \bot$ implies $M, w \models \varphi \land \psi > \bot$.*

The fourth condition is a restricted monotonicity constraint. It says that, when we strengthen an antecedent from X to $Y \subseteq X$, we must get a stronger hypothetical context $f(w, Y) \subseteq f(w, X)$, as long as the stronger antecedent is consistent with the hypothetical context determined by the weaker antecedent.

The fifth condition is also about the effect of strengthening an antecedent. It says that if v is one of the relevant X-worlds at w, and if v also satisfies a stronger proposition $Y \subseteq X$, then v is also one of the relevant Y-worlds at w.

Notice that conditions 4 and 5 jointly determine the effect of strengthening an antecedent in those cases in which the stronger antecedent is consistent with the hypothetical context for the weak one.

Proposition 7.
Let M obey 4 and 5. If $Y \subseteq X$ and $f(w, X) \cap Y \neq \emptyset$, then $f(w, Y) = f(w, X) \cap Y$.

Proof. Suppose $Y \subseteq X$ and $f(w, X) \cap Y \neq \emptyset$. By definition of selection function, $f(w, Y) \subseteq Y$, and by condition 4, $f(w, Y) \subseteq f(w, X)$. So, $f(w, Y) \subseteq f(w, X) \cap Y$. The converse inclusion is given by Condition 5.

Notice that, when we restrict ourselves to classical selection models, where $\leq$ is the identity, the above conditions pick out exactly the class of classical selection models which characterize Lewis's logic VC (see [14], §2.7). The logic of all intuitionistic minimal change models can thus be naturally regarded as an intuitionistic counterpart of VC. We will denote this logic as IVC.

Definition 4 (Logic IVC).
For $\Phi \cup \{\psi\} \subseteq \mathcal{L}^{>}$, we write $\Phi \models_{\mathsf{IVC}} \psi$ iff for any intuitionistic minimal change model M and any world w, if $M, w \models \varphi$ for all $\varphi \in \Phi$, then $M, w \models \psi$.

4 Axiomatization

In this section we describe a Hilbert-style system for the logic IVC, and show that it is sound. In the following sections we will show that it is also complete. The system has three groups of axioms: axioms for intuitionistic propositional logic; axioms that pertain to selection function semantics in general; and axioms that correspond to the minimal change conditions.

- Intuitionistic schemata:
  - $\varphi \to (\psi \to \varphi)$
  - $(\varphi \to (\psi \to \chi)) \to ((\varphi \to \psi) \to (\varphi \to \chi))$
  - $\varphi \to (\psi \to \varphi \wedge \psi)$
  - $\varphi \wedge \psi \to \varphi, \;\; \varphi \wedge \psi \to \psi$
  - $\varphi \to \varphi \vee \psi, \;\; \psi \to \varphi \vee \psi$
  - $(\varphi \to \chi) \to ((\psi \to \chi) \to (\varphi \vee \psi \to \chi))$
  - $\bot \to \varphi$
- Selection function schemata:
  - $\varphi > \varphi$
  - $(\varphi > \psi \wedge \chi) \leftrightarrow (\varphi > \psi) \wedge (\varphi > \chi)$
- Minimal change schemata:
  - $(\varphi > \psi) \to (\varphi \to \psi)$
  - $(\varphi \wedge \psi) \to (\varphi > \psi)$
  - $(\varphi > \bot) \to (\varphi \wedge \psi > \bot)$
  - $(\varphi > \chi) \to (\varphi > \neg\psi) \vee ((\varphi \wedge \psi) > \chi)$
  - $(\varphi \wedge \psi > \chi) \to (\varphi > (\psi \to \chi))$

The system has three inference rules: *modus ponens*, *replacement of equivalent antecedents*, and *replacement of equivalent consequents*:

$$\frac{\varphi \quad \varphi \to \psi}{\psi}\ (\mathsf{MP}) \qquad \frac{\varphi \leftrightarrow \psi}{(\varphi > \chi) \leftrightarrow (\psi > \chi)}\ (\mathsf{RCEA}) \qquad \frac{\varphi \leftrightarrow \psi}{(\chi > \varphi) \leftrightarrow (\chi > \psi)}\ (\mathsf{RCEC})$$

As usual in modal logic, some care is needed when defining derivability from a set of assumptions in the system. We define this as follows.

Definition 5. *For $\Phi \cup \{\psi\} \subseteq \mathcal{L}^{>}$, we write $\Phi \vdash_{\mathsf{IVC}} \psi$ to mean that there exist $\varphi_1, \ldots, \varphi_n \in \Phi$ such that $\varphi_1 \wedge \cdots \wedge \varphi_n \rightarrow \psi$ is derivable in the system above.*

A fact that will be useful below is that, in IVC, conditionals are right monotonic.

Proposition 8. *If $\psi \vdash_{\mathsf{IVC}} \chi$ then $\varphi > \psi \vdash_{\mathsf{IVC}} \varphi > \chi$.*

Proof. If $\psi \vdash_{\mathsf{IVC}} \chi$ then $\vdash_{\mathsf{IVC}} \psi \rightarrow \chi$, and therefore by intuitionistic reasoning, $\vdash_{\mathsf{IVC}} \psi \leftrightarrow \psi \wedge \chi$. By closure under replacement of equivalent consequents, we have $\vdash_{\mathsf{IVC}} (\varphi > \psi) \leftrightarrow (\varphi > \psi \wedge \chi)$. But we also have the axiom $(\varphi > \psi \wedge \chi) \leftrightarrow (\varphi > \psi) \wedge (\varphi > \chi)$. By intuitionistic reasoning it follows that $\vdash_{\mathsf{IVC}} (\varphi > \psi) \rightarrow (\varphi > \chi)$. Thus, $\varphi > \psi \vdash_{\mathsf{IVC}} \varphi > \chi$. □

Proposition 9 (Soundness). *If $\Phi \models_{\mathsf{IVC}} \psi$ then $\Phi \vdash_{\mathsf{IVC}} \psi$.*

Proof. As usual, the proof amounts to checking that the axioms of the system are valid, and that the inference rules preserve validity. We focus on the soundness of the minimal change schemata, since the other cases are straightforward. The soundness of the first three schemata is an immediate consequence of Propositions 3, 5, and 6. Consider the fourth minimal change schema. Suppose that $M, w \models \varphi > \chi$, i.e., $f(w, \varphi) \subseteq |\chi|$. We want to show that $M, w \models (\varphi \rightarrow \neg\psi) \vee (\varphi \wedge \psi > \chi)$. We distinguish two cases:

- Case 1: $f(w, \varphi) \cap |\psi| = \emptyset$. Take any $v \in f(w, \varphi)$. Since $f(w, \varphi)$ is upwards closed, for any successor $u \geq v$ we have $u \in f(w, \varphi)$, and therefore $u \notin |\psi|$. This means that $M, v \models \neg\psi$. Therefore, $M, w \models \varphi > \neg\psi$.
- Case 2: $f(w, \varphi) \cap |\psi| \neq \emptyset$. Then we have $|\varphi \wedge \psi| \subseteq |\varphi|$ and $f(w, \varphi) \cap |\varphi \wedge \psi| \neq \emptyset$. Therefore, Condition 4 implies $f(w, \varphi \wedge \psi) \subseteq f(w, \varphi)$, and since $f(w, \varphi) \subseteq |\chi|$ we have $M, w \models \varphi \wedge \psi > \chi$.

In both cases, $M, w \models (\varphi \rightarrow \neg\psi) \vee (\varphi \wedge \psi > \chi)$.

Finally, consider an instance of the schema $(\varphi \wedge \psi > \chi) \rightarrow (\varphi > (\psi \rightarrow \chi))$. Suppose $M, w \models \varphi \wedge \psi > \chi$. Then $f(w, \varphi \wedge \psi) \subseteq |\chi|$. We need to show that $w \models \varphi > (\psi \rightarrow \chi)$. So, take any $v \in f(w, \varphi)$. We want to show $v \models \psi \rightarrow \chi$. Consider any $u \geq v$ with $M, u \models \psi$. Since $f(w, \varphi)$ is upwards closed, $u \in f(w, \varphi)$. Thus, $u \in f(w, \varphi) \cap |\psi| = f(w, \varphi) \cap |\varphi \wedge \psi|$. By Condition 5, $f(w, \varphi) \cap |\varphi \wedge \psi| \subseteq f(w, \varphi \wedge \psi) \subseteq |\chi|$. Therefore, $u \in |\chi|$, which means that $M, u \models \chi$, as we wanted.

Notice that the proof makes crucial use of the upwards closure condition of $f(w, \varphi)$, which distinguishes our semantics from the one of Weiss [21]. It is not hard to show that, if $f(w, \varphi)$ is not required to be upwards closed, then the last two minimal change schemata are not sound in general.

5 Canonical model construction

In this section we define a canonical model for IVC, which will allow us to show the completeness of our proof system. As usual in intuitionistic logic, the model is based on consistent theories with the disjunction property.

Definition 6. *Let $\Gamma \subseteq \mathcal{L}^{>}$. We say that:*

- Γ *is an* IVC-theory *if for all* $\varphi \in \mathcal{L}^{>}$: $\Gamma \vdash_{IVC} \varphi$ *implies* $\varphi \in \Gamma$;
- Γ *is a* consistent IVC-theory *if* Γ *is an IVC-theory and* $\bot \notin \Gamma$;
- Γ *has the* disjunction property *if* $\varphi \vee \psi \in \Gamma$ *implies* $\varphi \in \Gamma$ *or* $\psi \in \Gamma$.

The next lemma, familiar in intuitionistic logic, says that there are enough consistent IVC-theories with the disjunction property to witness all non-entailments in IVC. The standard proof of the lemma is omitted (see, e.g., Lemma 11 in [4]).

Lemma 1. *If* $\Phi \not\vdash_{IVC} \psi$ *then there exists a consistent IVC-theory with the disjunction property* Γ *such that* $\Phi \subseteq \Gamma$ *and* $\psi \notin \Gamma$.

For an IVC-theory Γ, the set of counterfactual consequences of φ in Γ is:

- $\mathrm{Cn}_{\varphi}(\Gamma) := \{\psi \in \mathcal{L}^{>} \mid \varphi > \psi \in \Gamma\}$

The following feature of $\mathrm{Cn}_{\varphi}(\Gamma)$ will play an important role below.

Lemma 2. *If* Γ *is an IVC-theory, then so is* $Cn_{\varphi}(\Gamma)$.

Proof. Straightforward, using the right monotonicity of $>$ (Proposition 8).

Definition 7 (Canonical model).
The canonical model for IVC is the model $M^c_{IVC} = \langle W^c, \leq^c, \mathcal{A}^c, f^c, V^c\rangle$, *where:*

- W^c *is the set of consistent IVC-theories with the disjunction property;*
- $\Gamma \leq^c \Gamma' \iff \Gamma \subseteq \Gamma'$
- $\mathcal{A}^c = \{\widehat{\varphi} \mid \varphi \in \mathcal{L}^{>}\}$ *where* $\widehat{\varphi} := \{\Gamma \in W^c \mid \varphi \in \Gamma\}$
- $f^c(\Gamma, \widehat{\varphi}) = \{\Gamma' \in W^c \mid Cn_{\varphi}(\Gamma) \subseteq \Gamma'\}$
- $V^c(p) = \widehat{p}$

We need to make sure that f^c is well-defined, i.e., that if $\widehat{\varphi} = \widehat{\chi}$ then $f^c(\Gamma, \widehat{\varphi}) = f^c(\Gamma, \widehat{\chi})$. This is guaranteed by the following proposition.

Proposition 10. *If* $\widehat{\varphi} = \widehat{\chi}$, *then for all* $\Gamma \in W^c$: $Cn_{\varphi}(\Gamma) = Cn_{\chi}(\Gamma)$.

Proof. Suppose $\widehat{\varphi} = \widehat{\chi}$. First, note that this implies $\vdash_{\mathsf{IVC}} \varphi \leftrightarrow \chi$. For suppose not: then $\varphi \not\vdash_{\mathsf{IVC}} \chi$ or $\chi \not\vdash_{\mathsf{IVC}} \varphi$. Without loss of generality, suppose the former. By Lemma 1 there exists a theory $\Gamma \in W^c$ with $\varphi \in \Gamma$ and $\psi \notin \Gamma$, which means that $\Gamma \in \widehat{\varphi}$ but $\Gamma \notin \widehat{\chi}$, contrary to $\widehat{\varphi} = \widehat{\chi}$. So, $\vdash_{\mathsf{IVC}} \varphi \leftrightarrow \chi$. By replacement of equivalent antecedents, it follows that for any ψ, $\vdash_{\mathsf{IVC}} (\varphi > \psi) \leftrightarrow (\chi > \psi)$. Now take any $\Gamma \in W^c$. Since Γ is an IVC-theory, for any ψ we have $(\varphi > \psi) \in \Gamma \iff (\chi > \psi) \in \Gamma$. As this holds for all ψ, it follows that $\mathrm{Cn}_{\varphi}(\Gamma) = \mathrm{Cn}_{\varphi}(\Gamma)$.

The next proposition ensures that M^c_{IVC} satisfies all the conditions required by Definition 1.

Proposition 11. M^c_{IVC} *is an intuitionistic selection model.*

Proof. We need to check that all the conditions in Definition 1 are satisfied. Clearly, the relation $\subseteq$ is a partial order on W^c. Every element of $\mathcal{A}^c$ is upwards closed, and $V^c(p) \in \mathcal{A}^c$. Four conditions remains to be checked:

- Closure of $\mathcal{A}^c$ under logic. We show in detail the most interesting case, namely, closure of $\mathcal{A}^c$ under the operation corresponding to $>$. So, suppose $\widehat{\varphi}, \widehat{\chi} \in \mathcal{A}$; we need to show that $\{\Gamma \mid f^c(\Gamma, \widehat{\varphi}) \subseteq \widehat{\chi}\} \in \mathcal{A}^c$. This will follow if we can show that
$$\{\Gamma \mid f^c(\Gamma, \widehat{\varphi}) \subseteq \widehat{\chi}\} = \widehat{\varphi > \chi}$$
This amounts to the claim that, for $\Gamma \in W^c$: $f^c(\Gamma, \widehat{\varphi}) \subseteq \widehat{\chi} \iff \varphi > \chi \in \Gamma$. In one direction, suppose $\varphi > \chi \in \Gamma$. Then $\chi \in \mathrm{Cn}_\varphi(\Gamma)$. Therefore any $\Gamma' \in f^c(\Gamma, \widehat{\varphi})$ must contain χ, which means that $f^c(\Gamma, \widehat{\varphi}) \subseteq \widehat{\chi}$. Conversely, suppose $\varphi > \chi \notin \Gamma$. Then $\chi \notin \mathrm{Cn}_\varphi(\Gamma)$. By Lemma 2, $\mathrm{Cn}_\varphi(\Gamma) \nvdash_{\mathsf{IVC}} \chi$. Therefore, by Lemma 1 there is $\Gamma' \in W^c$ with $\mathrm{Cn}_\varphi(\Gamma) \subseteq \Gamma'$ and $\chi \notin \Gamma'$. Thus, $\Gamma' \in f^c(\Gamma, \widehat{\varphi})$ but $\Gamma' \notin \widehat{\chi}$, witnessing that $f^c(\Gamma, \widehat{\varphi}) \not\subseteq \widehat{\chi}$.
In a similar fashion, it is easy to prove that $\mathcal{A}^c$ is closed under intersection, union, and the operation corresponding to $\to$, since $\widehat{\varphi} \cup \widehat{\chi} = \widehat{\varphi \vee \chi}$, $\widehat{\varphi} \cap \widehat{\chi} = \widehat{\varphi \wedge \chi}$, and $\{\Gamma \in W^c \mid \widehat{\varphi} \cap \Gamma^{\uparrow} \subseteq \widehat{\psi}\} = \widehat{\varphi \to \chi}$.
- $f^c(\Gamma, \widehat{\varphi}) \subseteq \widehat{\varphi}$. Take $\Gamma' \in f^c(\Gamma, \widehat{\varphi})$. This means that $\mathrm{Cn}_\varphi(\Gamma) \subseteq \Gamma'$. By the axiom $\varphi > \varphi$ we have $\varphi \in \mathrm{Cn}_\varphi(\Gamma)$. Thus, $\varphi \in \Gamma'$, which shows that $\Gamma' \in \widehat{\varphi}$.
- $f^c(\Gamma, \widehat{\varphi})$ is upwards closed. This is clear since, if $\Gamma' \subseteq \Gamma''$ we have $\Gamma' \in f^c(\Gamma, \widehat{\varphi}) \iff \mathrm{Cn}_\varphi(\Gamma) \subseteq \Gamma' \Longrightarrow \mathrm{Cn}_\varphi(\Gamma) \subseteq \Gamma'' \iff \Gamma'' \in f^c(\Gamma, \widehat{\varphi})$.
- f^c is monotonic in the first coordinate. This is also clear: if $\Gamma \subseteq \Gamma'$ then $\mathrm{Cn}_\varphi(\Gamma) \subseteq \mathrm{Cn}_\varphi(\Gamma')$, and therefore $\Gamma'' \in f^c(\Gamma', \widehat{\varphi}) \iff \mathrm{Cn}_\varphi(\Gamma') \subseteq \Gamma'' \Longrightarrow \mathrm{Cn}_\varphi(\Gamma) \subseteq \Gamma'' \iff \Gamma'' \in f^c(\Gamma, \widehat{\varphi})$. □

Moreover, we can prove that M^c_L behaves in the way expected of a canonical model: satisfaction at a theory Γ amounts to membership in Γ.

Lemma 3 (Truth Lemma).
For any $\Gamma \in W^c$ and any $\varphi \in \mathcal{L}^>$: $M^c_{\mathsf{IVC}}, \Gamma \models \varphi \iff \varphi \in \Gamma$.

Proof. As usual, the proof is by induction on φ. We only give the inductive step for $\varphi = \chi > \psi$, since the other steps are the same as in the case of intuitionistic propositional logic.

For the right-to-left direction, suppose $\chi > \psi \in \Gamma$. Then $\psi \in \mathrm{Cn}_\chi(\Gamma)$, so $\psi \in \Gamma'$ for every $\Gamma' \in f^c(\Gamma, \widehat{\chi})$, by definition of $f^c(\Gamma, \widehat{\chi})$. By the induction hypothesis on ψ, this means that for every $\Gamma' \in f^c(\Gamma, \widehat{\chi})$ we have $M^c_{\mathsf{IVC}}, \Gamma' \models \psi$. Moreover, by the induction hypothesis on χ we have $\widehat{\chi} = |\chi|$. Thus, $M^c_{\mathsf{IVC}}, \Gamma \models \chi > \psi$.

For the converse direction, suppose $\chi > \psi \notin \Gamma$. Then $\psi \notin \mathrm{Cn}_\chi(\Gamma)$, which by Lemma 2 means that $\mathrm{Cn}_\chi(\Gamma) \nvdash_{\mathsf{IVC}} \psi$. By Lemma 1 there exists a theory $\Gamma' \in W^c$ such that (i) $\mathrm{Cn}_\chi(\Gamma) \subseteq \Gamma'$ and (ii) $\psi \notin \Gamma'$. By (i) we have $\Gamma' \in f^c(\Gamma, \widehat{\chi})$, and by (ii) and the induction hypothesis on ψ, $M^c_{\mathsf{IVC}}, \Gamma' \not\models \psi$. Moreover, by the induction hypothesis on χ we have $\widehat{\chi} = |\chi|$. Therefore, it is not the case that all worlds in $f^c(\Gamma, |\chi|)$ satisfy ψ, which means that $M^c_{\mathsf{IVC}}, \Gamma \not\models \chi > \psi$. □

This lemma implies that any IVC-invalid entailment can be falsified in M^c_{IVC}.

Proposition 12. *If $\Phi \not\vdash_{\mathsf{IVC}} \psi$, then there exists a world $\Gamma \in W^c$ such that $M^c_{\mathsf{IVC}}, \Gamma \models \varphi$ for all $\varphi \in \Phi$ but $M^c_{\mathsf{IVC}}, \Gamma \not\models \psi$.*

Proof. If $\Phi \not\vdash_{\mathsf{IVC}} \psi$, by Lemma 1 there is $\Gamma \in W^c$ s.t. $\Phi \subseteq \Gamma$ but $\psi \notin \Gamma$. By the truth-lemma, world Γ in M^c_{IVC} satisfies all formulas in Φ but not ψ. □

To show that our system is complete with respect to the logic of intuitionistic minimal change models, all that remains to be shown is the following.

Proposition 13. *M^c_{IVC} is an intuitionistic minimal change model.*

Proof. We will show that each of the five minimal-change axioms of the logic IVC yields one of the corresponding minimal change properties for M^c_{IVC}.

- Condition 1. Suppose $\Gamma \in \widehat{\varphi}$, which means that $\varphi \in \Gamma$. We want to show that $\Gamma \in f^c(\Gamma, \widehat{\varphi})$, which amounts to $\mathrm{Cn}_\varphi(\Gamma) \subseteq \Gamma$. Consider any $\psi \in \mathrm{Cn}_\varphi(\Gamma)$. This means that $\varphi > \psi \in \Gamma$. By the axiom $(\varphi > \psi) \to (\varphi \to \psi)$, also $\varphi \to \psi \in \Gamma$. Since $\varphi \in \Gamma$, it follows $\psi \in \Gamma$.
- Condition 2. Take a point $\Gamma \in \widehat{\varphi}$, which means that $\varphi \in \Gamma$. We want to show that $f^c(\Gamma, \widehat{\varphi}) \subseteq \Gamma^{\uparrow}$. So, take any $\Gamma' \in f^c(\Gamma, \widehat{\varphi})$. This means that $\mathrm{Cn}_\varphi(\Gamma) \subseteq \Gamma'$. We need to show that $\Gamma' \in \Gamma^{\uparrow}$, which amounts to $\Gamma \subseteq \Gamma'$. This will follow if we can show that $\Gamma \subseteq \mathrm{Cn}_\varphi(\Gamma)$. So, take any $\psi \in \Gamma$. Since $\varphi, \psi \in \Gamma$, also $\varphi \wedge \psi \in \Gamma$. By the axiom $(\varphi \wedge \psi) \to (\varphi > \psi)$, it follows that $\varphi > \psi \in \Gamma$, so $\psi \in \mathrm{Cn}_\varphi(\Gamma)$.
- Condition 3. Suppose $\widehat{\psi} \subseteq \widehat{\varphi}$. This implies $\psi \vdash_{\mathsf{IVC}} \varphi$: for otherwise, by Lemma 1 there would be a theory $\Gamma \in W^c$ such that $\psi \in \Gamma$ and $\varphi \notin \Gamma$; and then $\Gamma \in \widehat{\psi} - \widehat{\varphi}$, contrary to the inclusion $\widehat{\psi} \subseteq \widehat{\varphi}$.
  Now suppose that $f^c(\Gamma, \widehat{\varphi}) = \emptyset$. We want to show that also $f^c(\Gamma, \widehat{\psi}) = \emptyset$. First, notice that $f^c(\Gamma, \widehat{\varphi}) = \emptyset$ implies that $\mathrm{Cn}_\varphi(\Gamma) \vdash_{\mathsf{IVC}} \bot$. For otherwise, by Lemma 1, $\mathrm{Cn}_\varphi(\Gamma)$ could be extended to a world $\Gamma' \in W^c$, and then we would have $\Gamma' \in f^c(\Gamma, \widehat{\varphi})$.
  By Lemma 2, $\mathrm{Cn}_\varphi(\Gamma) \vdash_{\mathsf{IVC}} \bot$ implies $\bot \in \mathrm{Cn}_\varphi(\Gamma)$, that is, $\varphi > \bot \in \Gamma$. By the axiom $(\varphi > \bot) \to (\varphi \wedge \psi > \bot)$ we also have $\varphi \wedge \psi > \bot \in \Gamma$. Since $\psi \vdash_{\mathsf{IVC}} \varphi$, the counterfactual $\varphi \wedge \psi > \bot$ is inter-derivable with $\psi > \bot$. Therefore, $\psi > \bot \in \Gamma$, which implies that $\bot \in \mathrm{Cn}_\psi(\Gamma)$.
  Now for every $\Gamma' \in W^c$ we have $\bot \notin \Gamma'$, and therefore $\mathrm{Cn}_\psi(\Gamma) \not\subseteq \Gamma'$. This shows that $f^c(\Gamma, \varphi \wedge \psi) = \emptyset$.
- Condition 4. Suppose $\widehat{\psi} \subseteq \widehat{\varphi}$. As discussed in the previous point, this implies $\psi \vdash_{\mathsf{IVC}} \varphi$. Suppose moreover that $f^c(\Gamma, \widehat{\varphi}) \cap \widehat{\psi} \neq \emptyset$. This means that there is $\Gamma' \in W^c$ with $\mathrm{Cn}_\varphi(\Gamma) \cup \{\psi\} \subseteq \Gamma'$. This implies that $\neg\psi \notin \mathrm{Cn}_\varphi(\Gamma)$, since otherwise Γ' would not be consistent. Hence, $\varphi > \neg\psi \notin \Gamma$.
  We want to show that $f^c(\Gamma, \widehat{\psi}) \subseteq f^c(\Gamma, \widehat{\varphi})$. Given the definition of f^c, this will follow if we can show that $\mathrm{Cn}_\psi(\Gamma) \supseteq \mathrm{Cn}_\varphi(\Gamma)$. So, take $\chi \in \mathrm{Cn}_\varphi(\Gamma)$. This means that $\varphi > \chi \in \Gamma$. Since Γ is an IVC-theory, by the axiom $(\varphi > \chi) \to ((\varphi > \neg\psi) \vee (\varphi \wedge \psi > \chi))$, it follows that $(\varphi > \neg\psi) \vee (\varphi \wedge \psi > \chi) \in \Gamma$. Since Γ has the disjunction property, one of the disjuncts is in Γ. Since we already know that $\varphi > \neg\psi \notin \Gamma$, it follows that $\varphi \wedge \psi > \chi \in \Gamma$. Since

$\psi \vdash_{\mathsf{IVC}} \varphi$, the counterfactual $\varphi \wedge \psi > \chi$ is IVC-equivalent to $\psi > \chi$. Therefore, $\psi > \chi \in \Gamma$, which implies that $\chi \in \mathrm{Cn}_\psi(\Gamma)$. This proves the desired inclusion $\mathrm{Cn}_\psi(\Gamma) \supseteq \mathrm{Cn}_\varphi(\Gamma)$.

- Condition 5. Suppose $\widehat{\psi} \subseteq \widehat{\varphi}$, which implies $\psi \vdash_{\mathsf{IVC}} \varphi$. We want to show that $f^c(\Gamma, \widehat{\varphi}) \cap \widehat{\psi} \subseteq f^c(\Gamma, \widehat{\psi})$. So, take any $\Gamma' \in f(\Gamma, \widehat{\varphi}) \cap \widehat{\psi}$: this means that $\mathrm{Cn}_\varphi(\Gamma) \subseteq \Gamma'$ and $\psi \in \Gamma'$. We need to prove that $\Gamma' \subseteq f^c(\Gamma, \widehat{\psi})$, which amounts to showing that $\mathrm{Cn}_\psi(\Gamma) \subseteq \Gamma'$.
  So, suppose $\chi \in \mathrm{Cn}_\psi(\Gamma)$. This means that $\psi > \chi \in \Gamma$. Since $\psi \vdash_{\mathsf{IVC}} \varphi$, the counterfactual $\psi > \chi$ is inter-derivable with $\varphi \wedge \psi > \chi$, therefore also $\varphi \wedge \psi > \chi \in \Gamma$. By the axiom $(\varphi \wedge \psi > \chi) \to (\varphi > (\psi \to \chi))$, it follows that also $\varphi > (\psi \to \chi) \in \Gamma$. Thus, $\psi \to \chi \in \mathrm{Cn}_\varphi(\Gamma) \subseteq \Gamma'$. Since also $\psi \in \Gamma'$, it follows that $\chi \in \Gamma'$. This shows the required inclusion $\mathrm{Cn}_{\varphi \wedge \psi}(\Gamma) \subseteq \Gamma'$. □

Our main result is now a corollary of propositions 12 and 13.

Theorem 1 (Completeness). *If $\Phi \models_{\mathsf{IVC}} \psi$, then $\Phi \vdash_{\mathsf{IVC}} \psi$.*

6 Conclusion and outlook

In this paper we saw that it is possible to extend intuitionistic Kripke semantics in a natural way with a Lewisian counterfactual conditional. The resulting logic, IVC, is an intuitionistic counterpart of Lewis' VC: indeed, modulo the replacement of classical propositional logic by intuitionistic logic, the axioms for IVC are the same as the axioms for VC, provided the formulation of the latter is chosen in a suitable way. In particular, the rational monotonicity axiom should be formulated in a constructive form as $(\varphi > \chi) \to (\varphi > \neg\psi) \vee (\varphi \wedge \psi > \chi)$ and not, as more common in the literature, as $(\varphi > \chi) \wedge \neg(\varphi > \neg\chi) \to (\varphi \wedge \psi > \chi)$.

This work may be taken further in several directions. First, while we focused here on a specific set of constraints, determining a specific intuitionistic counterfactual logic IVC, it would be interesting to take a broader perspective, and study the correspondence between constraints on f and conditional axioms in a more general way. This may yield intuitionistic counterparts of other notable logics of counterfactuals, such as Stalnaker's logic C2. Second, we may study how counterfactuals can be added not just to intuitionistic logic, but to intermediate logics more generally. Technically, this could be done by placing constraints not just on the selction function f, but also on the intuitionistic accessibility relation $\leq$. Third, it would be interesting to study not only the operator $>$, which Lewis denotes by $\Box\!\!\rightarrow$, but also its dual, which Lewis denotes by $\Diamond\!\!\rightarrow$. Whereas in the classical case the two are inter-definable via negation, in the intuitionistic case they must be treated as two independent operators, just like $\Box$ and $\Diamond$ need to be treated both as primitives in intuitionistic modal logic [10, 18]. Finally, it would be interesting to look at the relevance of IVC for the analysis of counterfactuals in natural language. For instance, recent work [8] provided experimental evidence that antecedents of the form $\neg p \vee \neg q$ and $\neg(p \wedge q)$ do not make the same contribution to a counterfactual: sentences of the form $(\neg p \vee \neg q) > r$ and $\neg(p \wedge q) > r$

do not in general have the same truth value. This runs against the predictions of any intensional account based on classical logic, since any such account renders $(\neg p \vee \neg q) > r$ and $\neg(p \wedge q) > r$ equivalent. By contrast, since the relevant de Morgan law is invalid intuitionistically, this is perfectly compatible with IVC.

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