Conditionals: between language and reasoning

Class 2, part 1 - The meta-linguistic theory

April 30, 2019

Two classes of conditionals:

- Ontic (aka counterfactuals):

  (1) If Oswald hadn't shot Kennedy, someone else would have.

- Epistemic (aka indicatives):

  (2) If Oswald didn't shoot Kennedy, someone else did.

- In part 1 of the course, we will focus on ontic conditionals.
- How should we analyze such conditionals?

- In part 1 of the course, we will focus on ontic conditionals.
- How should we analyze such conditionals?
- The material analysis is a non-starter:

  (3) If I hadn't taught this course, there would have been a student uprising.

  the antecedent of (3) is false, but this does not make (3) is true.
- Sentences like (3) are typically used when A is presupposed to be false; yet, this does not make them trivial.

The meta-linguistic theory (Goodman 47, Mackie 62, Rescher 64)
$A > C$ is true, in case C follows from A

The meta-linguistic theory (Goodman 47, Mackie 62, Rescher 64)
$A > C$ is true, in case C follows from A

(4) If just now I had turned this bottle upside down,
water would have poured out on the table.

Idea: (4) is true because the conclusion that water pours on the table follows from the premise that I turn the bottle upside down.

The meta-linguistic theory (Goodman 47, Mackie 62, Rescher 64)
$A > C$ is true, in case C follows from A

(4) If just now I had turned this bottle upside down,
water would have poured out on the table.

Idea: (4) is true because the conclusion that water pours on the table follows from the premise that I turn the bottle upside down.

But follows in what sense?

Complication 1: relevant conditions

(5) If just now I had turned this bottle upside down, water would have poured out on the table.

Complication 1: relevant conditions

(5) If just now I had turned this bottle upside down,
water would have poured out on the table.

The consequent does not follow from the antecedent alone, but also from some facts about the actual world:

- The bottle is uncapped.

Complication 1: relevant conditions

(5) If just now I had turned this bottle upside down,
water would have poured out on the table.

The consequent does not follow from the antecedent alone, but also from some facts about the actual world:

- The bottle is uncapped.
- There is water in the bottle.

Complication 1: relevant conditions

(5) If just now I had turned this bottle upside down,
water would have poured out on the table.

The consequent does not follow from the antecedent alone, but also from some facts about the actual world:

- The bottle is uncapped.
- There is water in the bottle.
- The water in the bottle is in a liquid state.

Complication 1: relevant conditions

(5) If just now I had turned this bottle upside down, water would have poured out on the table.

The consequent does not follow from the antecedent alone, but also from some facts about the actual world:

- The bottle is uncapped.
- There is water in the bottle.
- The water in the bottle is in a liquid state.

These are called by Goodman the relevant conditions.

More examples

(6) If I had arrived at 12.15, I would have been 15 minutes late.

More examples

(6) If I had arrived at 12.15, I would have been 15 minutes late.

- Depends on the time of the appointment. Was it at 12?

More examples

(6) If I had arrived at 12.15, I would have been 15 minutes late.

- Depends on the time of the appointment. Was it at 12?

(7) If I had saved 1e per day last year, in total I would have saved 365e.

More examples

(6) If I had arrived at 12.15, I would have been 15 minutes late.

▶Depends on the time of the appointment. Was it at 12?

(7) If I had saved 1e per day last year, in total I would have saved 365e.

▶Depends on the year: true if regular, false if leap.

Conclusion

The truth of counterfactuals depends on certain facts in the world.

The problem of relevant conditions:

which facts are we allowed to take into account?

Complication 2: inference by laws

(8) If just now I had turned this bottle upside down,
water would have poured out of it.

- Consider the antecedent augmented with all the relevant facts:

(9)
a. I have turned the bottle upside down.
b. The bottle is uncapped.
c. There is water in the bottle.
d. The water in the bottle is in a liquid state.
e. ...

Complication 2: inference by laws

(8) If just now I had turned this bottle upside down,
water would have poured out of it.

- Consider the antecedent augmented with all the relevant facts:

  (9)
  a. I have turned the bottle upside down.
  b. The bottle is uncapped.
  c. There is water in the bottle.
  d. The water in the bottle is in a liquid state.
  e. . . .

- The consequent does not follow logically from these assumptions.
- Rather, it follows from them via certain laws.

Complication 2: inference by laws

(8) If just now I had turned this bottle upside down,
water would have poured out of it.

- Consider the antecedent augmented with all the relevant facts:

  (9)
  a. I have turned the bottle upside down.
  b. The bottle is uncapped.
  c. There is water in the bottle.
  d. The water in the bottle is in a liquid state.
  e. . . .

- The consequent does not follow logically from these assumptions.

- Rather, it follows from them via certain laws.

Problem of laws:
which statements can be used as laws?

The meta-linguistic theory

$$A > C \text{ is true} \iff A + R \overset{L}{\rightsquigarrow} C$$

Challenge

Clarify what propositions belong to R and L.

The meta-linguistic theory

$$A > C \text{ is true} \iff A + R \overset{L}{\rightsquigarrow} C$$

Challenge

Clarify what propositions belong to R and L.

We will focus on the first problem, which pertains strictly to logical semantics: given A and the laws L, what true facts are we allowed to use as assumptions?

The second problem—what constitutes a law—is less of a semantic problem, but a more general issue for philosophy of science, or for psychology.

The problem

- Let us denote by "$\rightsquigarrow$" derivability via a given set L of laws:

$$\Gamma \rightsquigarrow C \iff \Gamma, L \models C$$

- $A > C$ is true $\iff$ $A + R \rightsquigarrow C$
- Let $\mathcal{F}$ denote the set of true statements—let's call these the facts.
- Which set $R \subseteq \mathcal{F}$ can be used as relevant conditions?
- We may zoom in for simplicity on the case in which A and C are false.

Attempt 1: all facts

$R = \mathcal{F}$

Attempt 1: all facts
$R = \mathcal{F}$

Problem:

- Since A is false, $\neg A \in \mathcal{F}$.
- So $A + \mathcal{F} \rightsquigarrow C$ for any C
- $A > C$ is always true

Attempt 2: existential quantification

$A > C$ is true $\iff \exists R \subseteq \mathcal{F} : A + R \rightsquigarrow C$

Attempt 2: existential quantification

$A > C$ is true $\iff \exists R \subseteq \mathcal{F} : A + R \rightsquigarrow C$ Problem:

- taking $R = \mathcal{F}$, we have again $A + \mathcal{F} \rightsquigarrow C$ for any C
- again, $A > C$ is always true

Attempt 3: facts which are logically consistent with A
$R = \{B \in \mathcal{F} \mid A \wedge B \not\models \bot\}$

Attempt 3: facts which are logically consistent with A

$R = \{B \in \mathcal{F} \mid A \wedge B \not\models \bot\}$

Problem:

- There will be facts B which are logically consistent with the antecedent, but inconsistent with it on the basis of the laws.

(10) a. $A =$ The water in this bottle freezes
 b. $B =$ The water in this bottle is at 20° C
 c. Law: water that is at 20°C does not freeze.

- $A, B \rightsquigarrow \bot$. Since $B \in R$, also $A + R \rightsquigarrow \bot$.
- $A > C$ comes out true for any C, for instance:

(11) If the water in this bottle had frozen, it would have boiled.

Attempt 4: facts which are consistent with A on the basis of the laws

$R = \{B \in \mathcal{F} \mid A \wedge B \not\rightsquigarrow \bot\}$

Attempt 4: facts which are consistent with A on the basis of the laws

$R = \{B \in \mathcal{F} \mid A \wedge B \not\leadsto \bot\}$

Problem:

- Consider:

(12) If Alice had a sibling, ...

- Now consider:

(13) a. B_1 = Alice has no brothers
b. B_2 = Alice has no sisters

Attempt 4: facts which are consistent with A on the basis of the laws

$R = \{B \in \mathcal{F} \mid A \wedge B \not\rightsquigarrow \bot\}$

Problem:

- Consider:

(12) If Alice had a sibling, ...

- Now consider:

(13) a. B_1 = Alice has no brothers
b. B_2 = Alice has no sisters

- B_1 is fully consistent with A plus the laws, so $B_1 \in R$. Similarly, $B_2 \in R$.

Attempt 4: facts which are consistent with A on the basis of the laws
$R = \{B \in \mathcal{F} \mid A \wedge B \not\rightsquigarrow \bot\}$

Problem:

- Consider:

  (12) If Alice had a sibling, ...

- Now consider:

  (13) a. $B_1 =$ Alice has no brothers
  b. $B_2 =$ Alice has no sisters

- B_1 is fully consistent with A plus the laws, so $B_1 \in R$. Similarly, $B_2 \in R$.
- But $A, B_1, B_2 \models \bot$, so a fortiori $A + R \rightsquigarrow \bot$

Attempt 4: facts which are consistent with A on the basis of the laws

$R = \{B \in \mathcal{F} \mid A \wedge B \not\rightsquigarrow \bot\}$

Problem:

- Consider:

  (12) If Alice had a sibling, ...

- Now consider:

  (13) a. B_1 = Alice has no brothers
  b. B_2 = Alice has no sisters

- B_1 is fully consistent with A plus the laws, so $B_1 \in R$. Similarly, $B_2 \in R$.
- But $A, B_1, B_2 \models \bot$, so a fortiori $A + R \rightsquigarrow \bot$
- So again $A > C$ comes out true for all C.

Attempt 5: existential quantification over antecedent compatible sets

Let us say that R is compatible with A if $A + R \not\rightsquigarrow \bot$

$$A > C \text{ is true} \iff \exists R \subseteq \mathcal{F} : R \text{ compatible with } A \text{ and } A + R \rightsquigarrow C$$

Attempt 5: existential quantification over antecedent compatible sets

Let us say that R is compatible with A if $A + R \not\rightsquigarrow \bot$

$$A > C \text{ is true} \iff \exists R \subseteq \mathcal{F} : R \text{ compatible with } A \text{ and } A + R \rightsquigarrow C$$

Problem:

- Consider:

(14) If Alice had been in Carolina, ...

(15) a. $B_1 =$ Alice is not in North Carolina
b. $B_2 =$ Alice is not in South Carolina

Attempt 5: existential quantification over antecedent compatible sets

Let us say that R is compatible with A if $A + R \not\rightsquigarrow \bot$

$$A > C \text{ is true} \iff \exists R \subseteq \mathcal{F} : R \text{ compatible with } A \text{ and } A + R \rightsquigarrow C$$

Problem:

- Consider:

(14) If Alice had been in Carolina, . . .

(15) a. $B_1 =$ Alice is not in North Carolina
b. $B_2 =$ Alice is not in South Carolina

- Now $R_1 = \{B_1\}$ is compatible with A and $A + R_1 \rightsquigarrow$ Alice is in SC. So, (16) is true:

(16) If Alice had been in Carolina, she would have been in SC.

Attempt 5: existential quantification over antecedent compatible sets

Let us say that R is compatible with A if $A + R \not\rightsquigarrow \bot$

$$A > C \text{ is true} \iff \exists R \subseteq \mathcal{F} : R \text{ compatible with } A \text{ and } A + R \rightsquigarrow C$$

Problem:

- Consider:

(14) If Alice had been in Carolina, ...

(15) a. $B_1 =$ Alice is not in North Carolina
b. $B_2 =$ Alice is not in South Carolina

- Now $R_1 = \{B_1\}$ is compatible with A and $A + R_1 \rightsquigarrow$ Alice is in SC. So, (16) is true:

(16) If Alice had been in Carolina, she would have been in SC.

- Similarly, since $\{B_2\}$ is compatible with A, (17) is true:

(17) If Alice had been in Carolina, she would have been in NC.

Attempt 5: existential quantification over antecedent compatible sets

Let us say that R is compatible with A if $A + R \not\rightsquigarrow \bot$

$$A > C \text{ is true} \iff \exists R \subseteq \mathcal{F} : R \text{ compatible with } A \text{ and } A + R \rightsquigarrow C$$

Problem:

- Consider:

(14) If Alice had been in Carolina, . . .

(15) a. B_1 = Alice is not in North Carolina
b. B_2 = Alice is not in South Carolina

- Now $R_1 = \{B_1\}$ is compatible with A and $A + R_1 \rightsquigarrow$ Alice is in SC. So, (16) is true:

(16) If Alice had been in Carolina, she would have been in SC.

- Similarly, since $\{B_2\}$ is compatible with A, (17) is true:

(17) If Alice had been in Carolina, she would have been in NC.

- But this seems wrong: (16) and (17) can't both be true.

Attempt 6: some antecedent-compatible sets lead to C, and none lead to $\neg C$

$$A > C \text{ is true} \iff \begin{array}{l} \exists R \subseteq \mathcal{F} : R \text{ compatible with } A \text{ and } A + R \leadsto C \text{ and} \\ \neg \exists R \subseteq \mathcal{F} : R \text{ compatible with } A \text{ and } A + R \leadsto \neg C \end{array}$$

Attempt 6: some antecedent-compatible sets lead to C, and none lead to $\neg C$

$$A > C \text{ is true} \iff \begin{array}{l} \exists R \subseteq \mathcal{F} : R \text{ compatible with } A \text{ and } A + R \rightsquigarrow C \text{ and} \\ \neg\exists R \subseteq \mathcal{F} : R \text{ compatible with } A \text{ and } A + R \rightsquigarrow \neg C \end{array}$$

Problem:

- Suppose $A \not\rightsquigarrow C$ (which is usually the case, as we saw).

Attempt 6: some antecedent-compatible sets lead to C, and none lead to $\neg C$

$$A > C \text{ is true} \iff \begin{array}{l} \exists R \subseteq \mathcal{F} : R \text{ compatible with } A \text{ and } A + R \rightsquigarrow C \text{ and} \\ \neg\exists R \subseteq \mathcal{F} : R \text{ compatible with } A \text{ and } A + R \rightsquigarrow \neg C \end{array}$$

Problem:

- Suppose $A \not\rightsquigarrow C$ (which is usually the case, as we saw).
- Then $A, \neg C \not\rightsquigarrow \bot$.

Attempt 6: some antecedent-compatible sets lead to C, and none lead to $\neg C$

$$A > C \text{ is true} \iff \begin{array}{l} \exists R \subseteq \mathcal{F} : R \text{ compatible with } A \text{ and } A + R \rightsquigarrow C \text{ and} \\ \neg\exists R \subseteq \mathcal{F} : R \text{ compatible with } A \text{ and } A + R \rightsquigarrow \neg C \end{array}$$

Problem:

- Suppose $A \not\rightsquigarrow C$ (which is usually the case, as we saw).
- Then $A, \neg C \not\rightsquigarrow \bot$.
- Then $R = \{\neg C\}$ is compatible with A and $A + R \rightsquigarrow \neg C$.

Attempt 6: some antecedent-compatible sets lead to C, and none lead to $\neg C$

$$A > C \text{ is true} \iff \begin{array}{l} \exists R \subseteq \mathcal{F} : R \text{ compatible with } A \text{ and } A + R \rightsquigarrow C \text{ and} \\ \neg\exists R \subseteq \mathcal{F} : R \text{ compatible with } A \text{ and } A + R \rightsquigarrow \neg C \end{array}$$

Problem:

- Suppose $A \not\rightsquigarrow C$ (which is usually the case, as we saw).
- Then $A, \neg C \not\rightsquigarrow \bot$.
- Then $R = \{\neg C\}$ is compatible with A and $A + R \rightsquigarrow \neg C$.
- So the second condition above is not true.
- Hence, $A > C$ is not true.

Attempt 6: some antecedent-compatible sets lead to C, and none lead to $\neg C$

$$A > C \text{ is true} \iff \begin{array}{l} \exists R \subseteq \mathcal{F} : R \text{ compatible with } A \text{ and } A + R \rightsquigarrow C \text{ and} \\ \neg\exists R \subseteq \mathcal{F} : R \text{ compatible with } A \text{ and } A + R \rightsquigarrow \neg C \end{array}$$

Problem:

- Suppose $A \not\rightsquigarrow C$ (which is usually the case, as we saw).
- Then $A, \neg C \not\rightsquigarrow \bot$.
- Then $R = \{\neg C\}$ is compatible with A and $A + R \rightsquigarrow \neg C$.
- So the second condition above is not true.
- Hence, $A > C$ is not true.
- Thus, this proposal would boil down to: $A > C$ true $\iff$ $A \rightsquigarrow C$
  Relevant conditions would play no role.

After a few more attempts, ever more baroque and still unsuccessful, Goodman ends up with the following:

Co-tenability
$R = \{B \in \mathcal{F} \mid \text{ it is not true that } A > \neg B\}$

After a few more attempts, ever more baroque and still unsuccessful, Goodman ends up with the following:

Co-tenability
$R = \{B \in \mathcal{F} \mid \text{ it is not true that } A > \neg B\}$

Problem:

- Circularity!
- The truth-conditions for $>$ appeal to a definition of relevant conditions; but this notion is in turn defined in terms of the truth-conditions for $>$.
- In order to determine the truth of any given counterfactual conditional, we would first have to determine the truth other such conditionals.
- "*Though unwilling to accept this conclusion, I do not at present see any way of meeting the difficulty.*"

How to react to this situation?

How to react to this situation?
Two responses:

Give up on precise truth-conditions (Stalnaker 68, Lewis 73)
We might not be able to specify exact truth-conditions for counterfactuals but we might be able to specify the math. form of these truth-conditions well enough to at least characterize a satisfactory logic of counterfactuals.

How to react to this situation?
Two responses:

Give up on precise truth-conditions (Stalnaker 68, Lewis 73)
We might not be able to specify exact truth-conditions for counterfactuals but we might be able to specify the math. form of these truth-conditions well enough to at least characterize a satisfactory logic of counterfactuals.

Embrace causal notions (Pearl 00, Schulz 11, Briggs 12, ...)
Provide tools to model causal connections and causal reasoning and use them to provide a non-circular implementation of the meta-linguistic theory.