# Questions and Dependence in Logic

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MCMP — 24.04.2020

## Course webpage

- www.ivanociardelli.altervista.org/questions-in-logic-20
- Material, weekly readings, exercises
- Course-related announcements

## Exercise session

- Wed 11–12, starting on May 6th.
- Work on the exercises at home, correction in class.
- Please do attend the exercise session!

## Assessment

- Unknown: will it be possible to hold a final exam?
- Yes $\rightsquigarrow$ Final exam, exercises similar to those in the exercise sessions.
- No $\rightsquigarrow$ To be decided: either take home exam or final essay.

## Topic

Some recent, interconnected, developments in logic.

- Inquisitive logic (main focus)
- Dependence logic
- Logics of epistemic vocabulary

} information-based approaches to logic

### Inquisitive logic

Extend the scope of logic to cover not only statements, but also questions.

Two main aims in the course:

### Conceptual

Bring out the relevance of questions for the field of logic,
and discuss how logical notions can be meaningfully applied to questions.

- Why are questions interesting from the perspective of a logician?
- How to make sense of logical consequence for questions?
- How to make sense of proofs in which we assume or conclude a question?

### Mathematical

Explore the mathematical theory of logic in this more general setting.

- Can standard logical operators ($\wedge$, $\forall$, ...) be extended to questions?
- What are the meta-theoretic properties of inquisitive logics?
- Can we give deduction systems for inferences with questions?

Research program currently active in various locations:

- Amsterdam (Roelofsen, van Gessel, Grilletti, Bezhanishvili)
- Berkeley (Holliday)
- Darmstadt (Otto, Meißner)
- Delft (Greco, Palmigiano)
- Helsinki (Yang, Barbero, Quadrellaro)
- Munich (Ciardelli)
- New York (Champollion)
- Paris (Mascarenhas)
- Prague (Punčochář)
- Sapporo (Sano)
- Utrecht (Iemhoff)

Today:

- Motivate the enterprise (slides)
- Foundations of the theory (whiteboard)

## Introduction

Traditionally, logic is concerned with relations between sentences which hold due to their logical form. For instance, (1a) and (1b) jointly entail (1c).

(1) a. All students passed the test.
b. Alice is a student.
c. Alice passed the test.

But it focuses on a particular type of sentences, namely, statements.

| | | | |
|---|---|---|---|
| (2) | a. | All students passed the test. | Statement |
| | b. | Which students passed the test? | Question |
| | c. | Students, make sure you pass the test! | Imperative |

## Why this restriction?

1. Logic is traditionally concerned with the correctness of arguments/proofs, and arguments/proofs consist of statements.

   (3) a. All students passed the test.
   b. Alice is a student.
   c. Therefore, Alice passed the test.

2. The central focus of logic is the relation of entailment/logical consequence. That is standardly understood as necessary preservation of truth.

   The notion of truth seems applicable to statements, but not to questions.

   (4) Munich is located in India. $\rightsquigarrow$ False

   (5) Where is Munich located? $\rightsquigarrow$ True or false?

## Beyond statements

Nevertheless, we'll see that it is possible to go beyond this traditional scope, and to bring questions within the purview of logic.

What is gained by doing so?

1. a more general account of logical connectives, quantifiers, modalities;
2. new, interesting logical notions emerge as cases of logical entailment;
3. a logical analysis of new modal notions comes within reach.

## Motivation 1

Logic is supposed to provide an analysis of logical items in language: connectives (and, if), quantifiers (every), modalities (know).

Logical items apply not just to statements, but also to questions:

(6) a. Alice rented a car and she booked a hotel.
b. What kind of car did Alice rent, and which hotel did she book?

(7) a. If Alice wins a free trip, she'll go to Athens.
b. If Alice wins a free trip, where will she go?

(8) a. Every student read a book.
b. What book did every student read?

(9) a. Bob knows that Alice lives in London.
b. Bob knows where Alice lives.

The standard account of these items only captures their role in the a-sentences. We'd like a more general theory that explains how they work across these pairs.

## Motivation 2

Beyond entailment and consistency, there are other important logical relations that have received less attention. One is dependency.

## Example

A die was tossed. The outcome, $x$, has not yet been revealed.
Then an informant tells us that the outcome is not 5 or 6.

The value of $x$ is determined by (i) whether $x$ is even (ii) whether $x$ is prime.

$$\begin{array}{rcl} Even(x), Prime(x) & \rightsquigarrow & x = 2 \\ \neg Even(x), Prime(x) & \rightsquigarrow & x = 3 \end{array} \qquad \begin{array}{rcl} Even(x), \neg Prime(x) & \rightsquigarrow & x = 4 \\ \neg Even(x), \neg Prime(x) & \rightsquigarrow & x = 1 \end{array}$$

$Q$ : what $x$ is

is determined by

$R_1$ : whether $x$ is even

$R_2$ : whether $x$ is prime

I will refer to this relation as dependency.

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What the relation amounts to the following:
in the given context, settling $R_1$ and $R_2$ implies settling $Q$.

Dependency is an important and quite ubiquitous relation,
which plays a key role in many different contexts.

## The relevance of dependence: Database Theory

| Student ID | ECTS winter | ECTS summer | ECTS year | Can graduate? |
|---|---|---|---|---|
| 0201 | 30 | 30 | 60 | yes |
| 0224 | 30 | 10 | 40 | no |
| 0233 | 10 | 30 | 40 | no |
| 0305 | 25 | 40 | 65 | yes |

Here are some dependencies that should hold:

- ECTS winter and ECTS summer jointly determine ECTS year
- ECTS year determines the Boolean field "Can graduate"
- Student ID determines any other field

## The relevance of dependencies: scientific theories

The predictive power of a theory $T$ lies in the dependencies that it yields.

$T$ is predictive of a question $Q$ given questions $R_1, \ldots, R_n$ iff in the context of $T$, $Q$ is completely determined by $R_1, \ldots, R_n$.

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## Example

$T =$ classical mechanics. A particle moving within a fixed forced field:

$$\left.\begin{array}{c}\text{position at } t_0\\ \text{velocity at } t_0\\ \text{mass}\end{array}\right\} \overset{\text{Determine}}{\Longrightarrow} \text{position at } t_1$$

## Dependency = question entailment

Once logic is extended to cover questions, dependency turns out to be a facet of the fundamental logical notion of entailment.

- This is not just a neat theoretical insight, but has concrete repercussions: dependency can be treated with the tools that logicians use for entailment.
- An example: to track entailment, logicians develop proof systems.
- If dependency is a case of entailment, then we can prove the validity of certain dependencies in a proof system equipped with questions.
- So, as we'll see, it does make sense to make inferences with questions. In fact, questions are powerful tools for inference.

## Motivation 3

Statements can occur embedded within intensional operators.

(10) Smith will be elected.

(11) a. I know that Smith will be elected.
b. It is possible at this stage that Smith will be elected.
c. The party has a strategy to ensure that Smith will be elected.

- The truth-cond. of (11a-c) depend on the proposition expressed by (10).
- In modal logic, we can formally model propositions as sets of worlds.
- That allows us to analyze many modal notions, such as knowledge, historical possibility, strategic ability..
- Often, these analyses lead to interesting logics of these notions.

Similarly, questions too can be embedded under intensional operators.

(12) Who will be elected?

(13) a. I wonder/don't care who will be elected.
b. The success of the plan depends on who will be elected.
c. The party controls who will be elected.

- These sentences illustrate several question-oriented modal notions: wondering, caring, historical dependence, strategic control.
- In order to give a formal analysis of these notions, we first need a semantic analysis of the complement "who gets elected".
- Once a question semantics is available, we can define operators on them. This makes it possible to give analyses of question-oriented modal notions.
- This broadens the scope of modal logic, bringing within its purview notions such as wondering, historical dependence, and strategic control.

Extend logic to questions.

- Why? ✓

Extend logic to questions.

- Why? ✓
- How? $\rightsquigarrow$ next